6.S965
Digital Systems Laboratory II

Lecture 10:

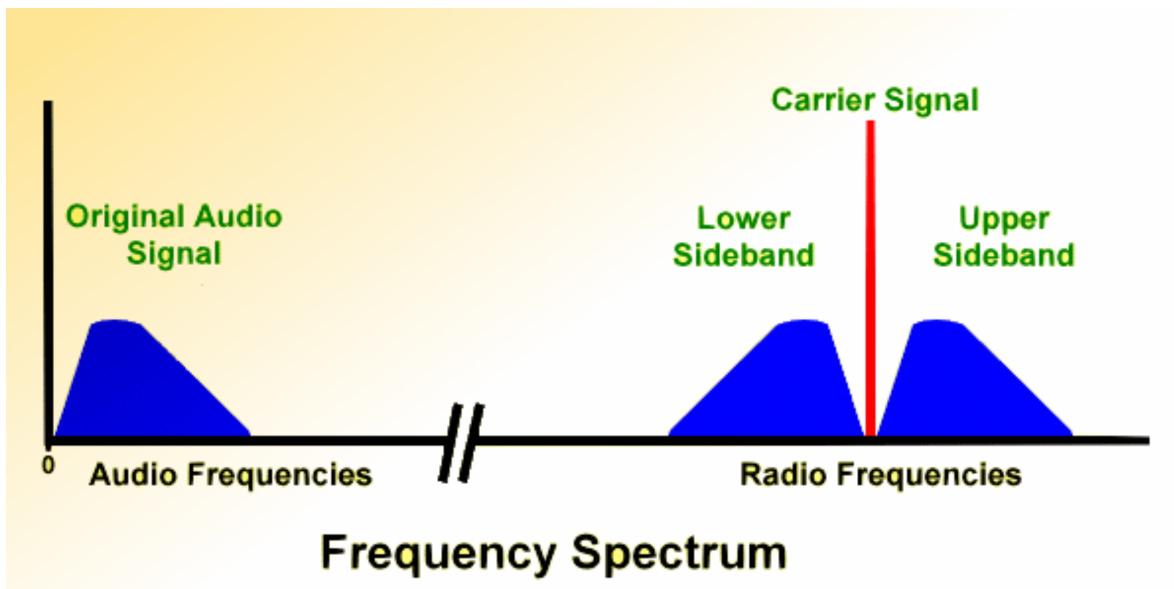
IQ 2

Admin

- Lab/Week 5 is out….no disasters so far, but please let me know!!!
- Grading should be *mostly* caught up

Why Is/Was AM easier to demodulate?

- Last week I mentioned that AM with its redundant sidebands of information and carrier was easier to demodulate than just the bare-minimum transferred of one sideband



AM

- This is an easy signal to make since it arises naturally from the multiplication

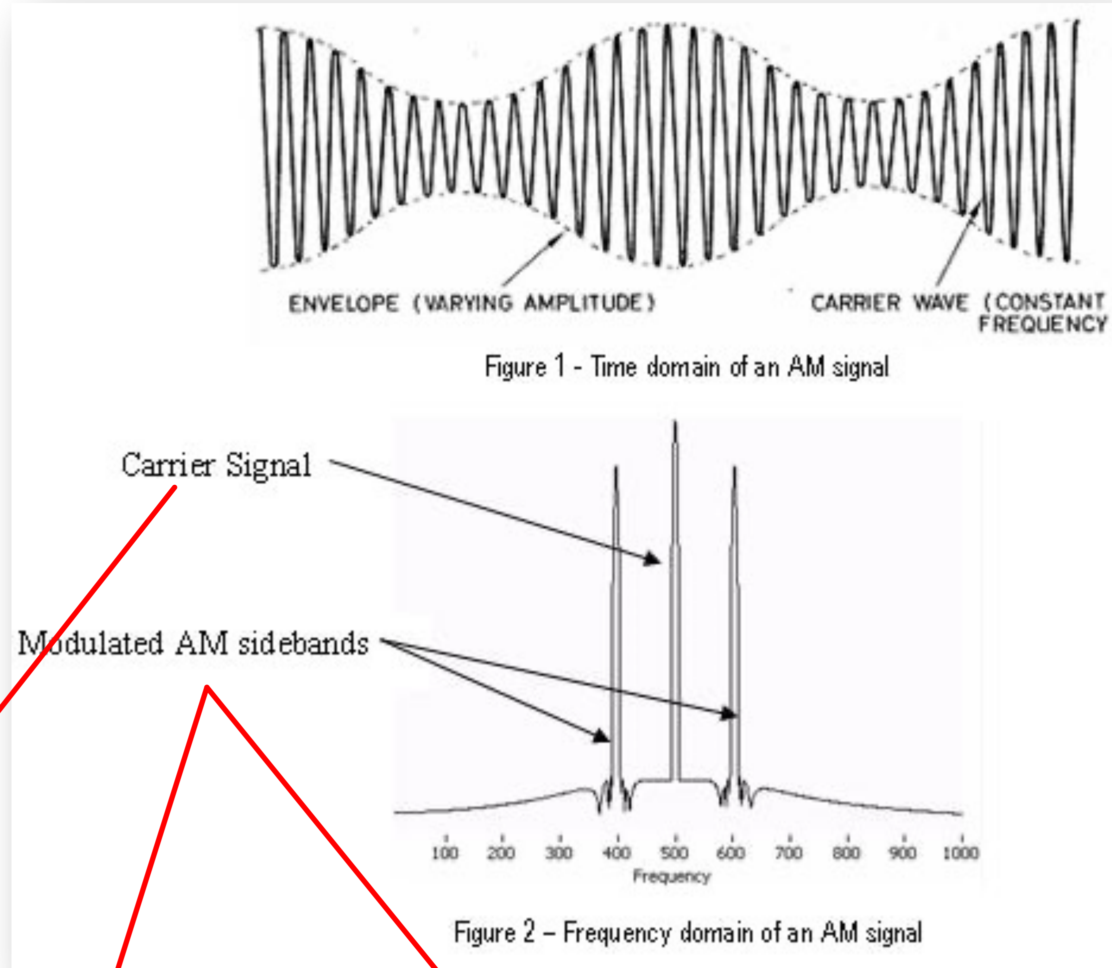


Figure 1 - Time domain of an AM signal

Figure 2 – Frequency domain of an AM signal

$$v_t(t) = A_c \cos(\omega_c t) + \frac{1}{2} \cdot \left(\cos\left((\omega_c - \omega_m)t\right) + \cos\left((\omega_c + \omega_m)t\right)\right)$$

https://knowledge.ni.com/KnowledgeArticleDetails?id=kA03q000000YGEbCAO&l=en-US

The Math of AM Demodulation

- If this is what we capture out of the sky:

$$\boldsymbol{v_t(t) = A_c \cos(\omega_c t) + \frac{1}{2} \cdot \left(\cos\left((\omega_c + \omega_m)t\right) + \cos\left((\omega_c - \omega_m)t\right)\right)}$$

- We want to have some sort of circuit allow us to perform operations that will let us get back to a signal that looks like this:

$$v_{rec}(t) \propto A_m \cos(\omega_m t)$$

- If we can do that, we're golden…but the only way to do that is to be able to use trig identities again:

$$\cos(a) \cdot \cos(b) = \frac{\cos(a-b) + \cos(a+b)}{2}$$

The Math of AM Demodulation

- If on the reception side we could somehow get this to happen:

$$\cos(\omega_c t) \cdot \cos\big((\omega_c + \omega_m)t\big)$$

- The result would be a term that is now only the modulation signal again (our information):

$$\propto \cos\Big(\big(\omega_c - (\omega_c + \omega_m)\big)t\Big) = \cos(\omega_m t)\ !$$

- But that requires ***multiplication of two electrical signals***…something you can't do with a linear system…need a multiplier

The Math of AM Demodulation

- If we could somehow take the *square* of our incoming AM signal, the FOIL-ing of the resultant math would have a term in there that accomplishes this!

$$v_t(t)^2 = v_t(t) \cdot v_t(t)$$

$$= \left(A_c \cos(\omega_c t) + \frac{1}{2} \cdot \left(\cos\left((\omega_c + \omega_m)t\right) + \cos\left((\omega_c - \omega_m)t\right)\right) \right)$$
$$\cdot \left(A_c \cos(\omega_c t) + \frac{1}{2} \cdot \left(\cos\left((\omega_c + \omega_m)t\right) + \cos\left((\omega_c - \omega_m)t\right)\right) \right)$$

- So we need a device that will square for us!!!
- That's annoying…not only do we want a non-linear device, but we need a device that is a particular type of non-linear.

Taylor Series to Rescue

- Think about a Taylor Series
- Model some device described by function $f(v)$ around some point V
- That function can be expressed as:

$$\sum_{k=0}^{n} \frac{f^{(k)}(v)}{k!}(v - V)^k$$

$$= f(V) + f'(V)(v - V) + \frac{f''(V)}{2}(v - V)^2 + \cdots$$

0th order term **1st order term** **2nd order term** **additional order terms**

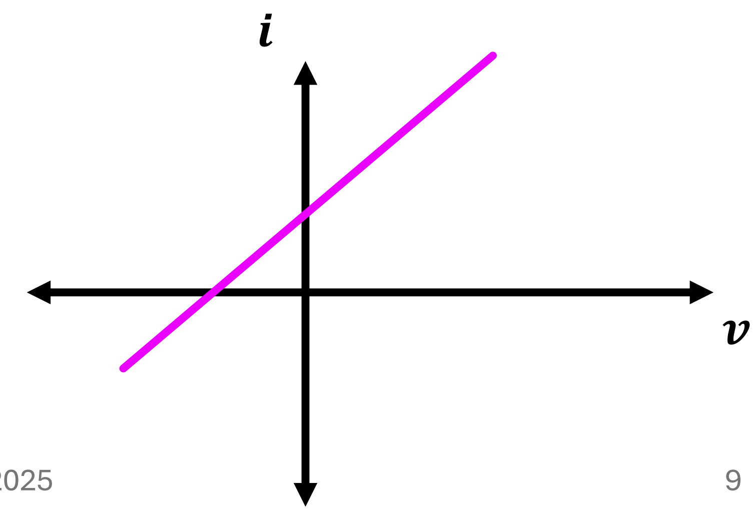
Electronics are Boring if you only have Linear Devices

- Linear Devices will expand at most to something like this:

$$= f(V) + f'(V)(v - V) + \frac{f''(V)}{2}(v - V)^2 + \cdots$$

Not function of V (just a constant)

No second order term



A Non-Linear Device

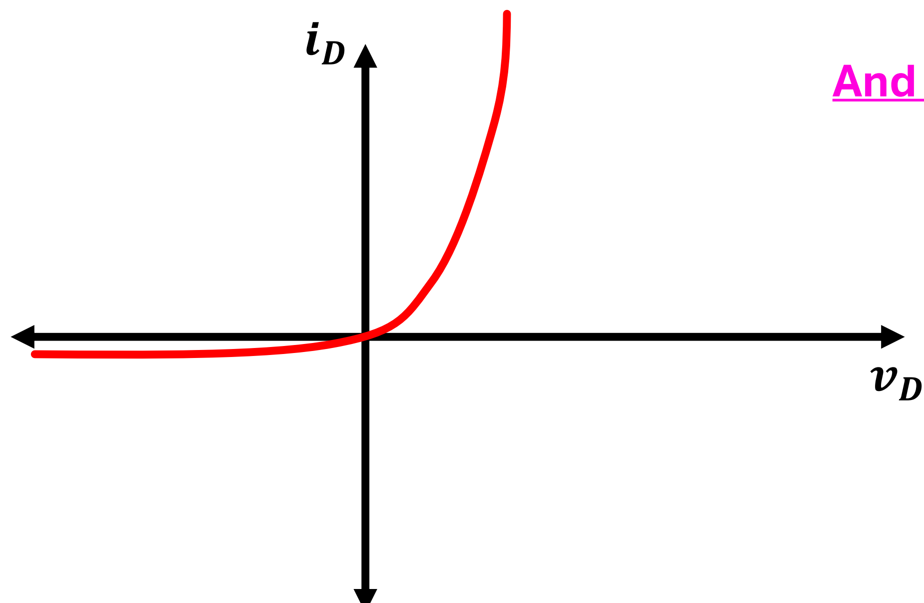
- Non-linear devices will have that and more potentially

$$= f(v) + f'(V)(v - V) + \frac{f''(V)}{2}(v - V)^2 + \cdots$$

Non-constant scalar

Gives us multiplication or in this case a squaring!

And potentially other crap



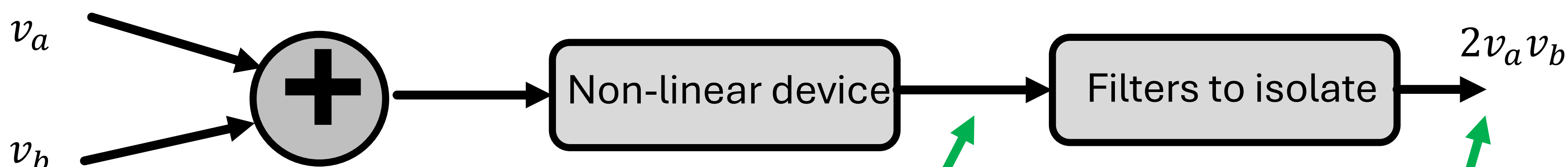
Conclusion

- Find a device with ***<u>reliable high-frequency</u>*** non-linearity
- Even if not a pure multiplier or squarer or whatever, if it is non-linear it will likely have some sort of squaring action due to its underlying Taylor Series Expansion!***
- Since multiplication of sinusoids results in sinusoids of differing frequencies, we could then use filters to block all the residue terms that we don't care about and isolate only the sinusoids we want (the ones containing information)

***The more square-like the better in the case of modulation, but it isn't a deal breaker if it departs from that

Both **Modulation** and Demodulation Benefit

- If we can produce good nonlinear devices, we can easily modulate as well as demodulate:
- On the transmit side you could do:



$$\frac{f''(V)}{2}(v_a + v_b - V)^2$$

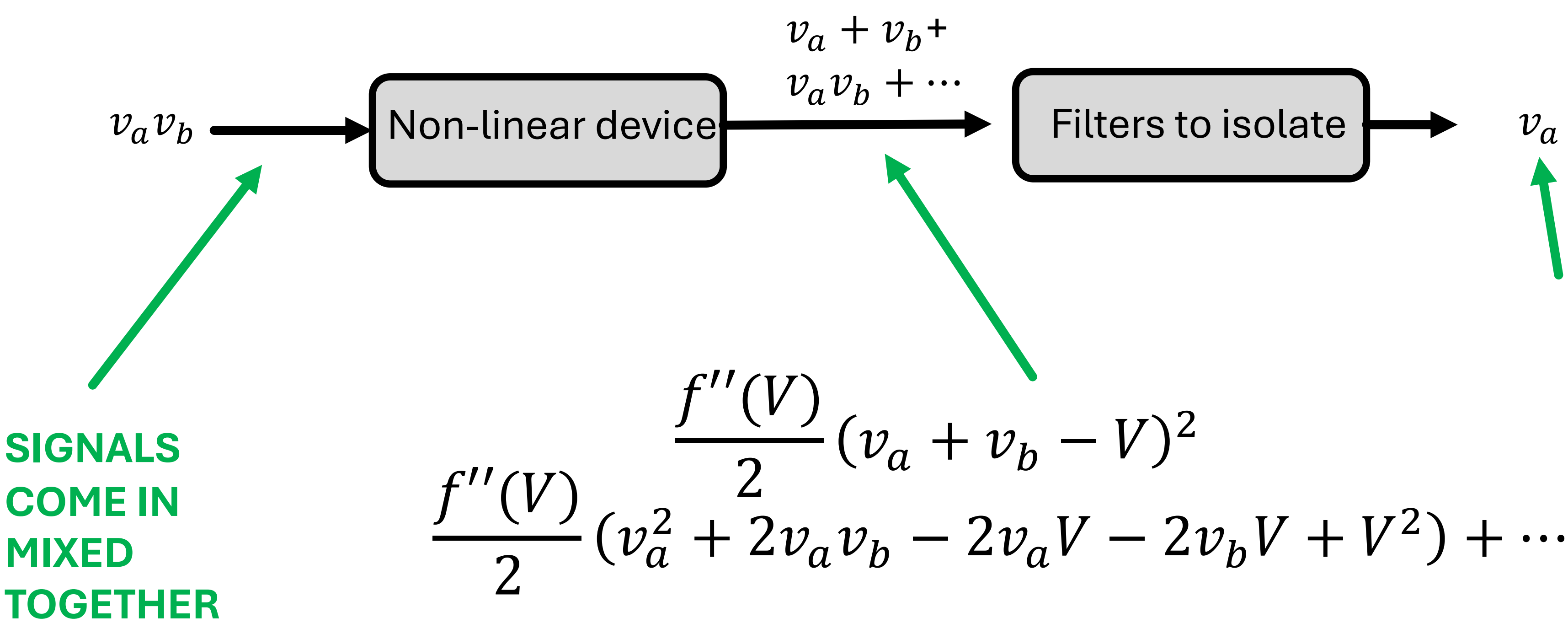
$$\frac{f''(V)}{2}(v_a^2 + 2v_a v_b - 2v_a V - 2v_b V + V^2) + \cdots$$

CAN ADD TWO SIGNALS WITH LINEAR ELEMENTS (RESITORS)

MULTIPLICATION of TWO SIGNALS!!

Both Modulation and **Demodulation** Benefit

- If we can produce good nonlinear devices, we can easily modulate as well as demodulate:
- On the receive side you could do:



$$\frac{f''(V)}{2}(v_a + v_b - V)^2$$

$$\frac{f''(V)}{2}(v_a^2 + 2v_a v_b - 2v_a V - 2v_b V + V^2) + \cdots$$

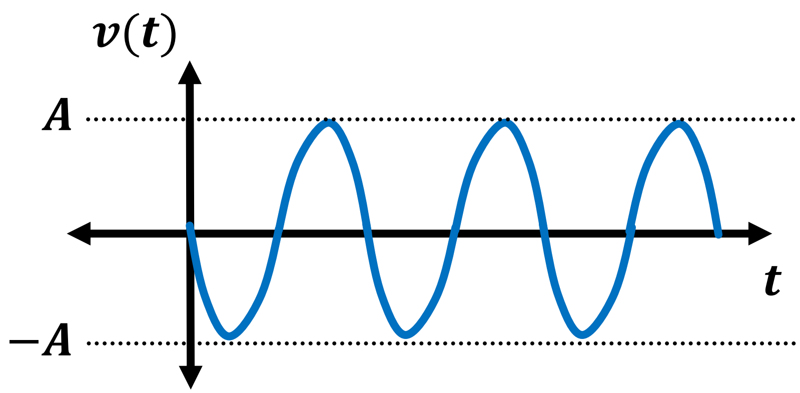
So 130 years ago people experimented

- All you needed was something that was reliably non-linear at high frequencies and you could get some modulation/demodulation

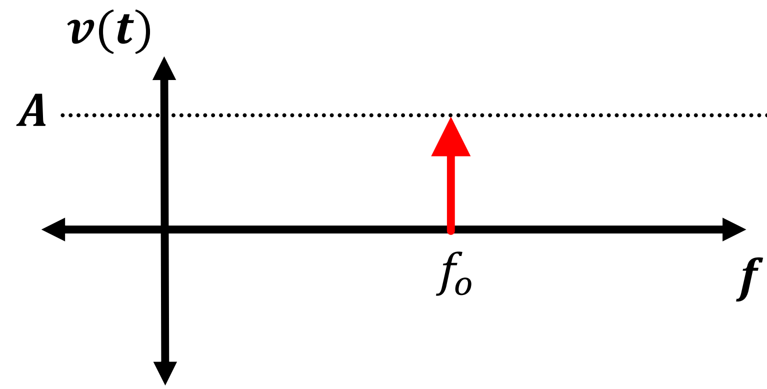
- One particular non-linearity started to appear quite a bit: rectification

Fourier Theory:

Incoming signal of $v_s(t) = A\sin(2\pi f_o t)$



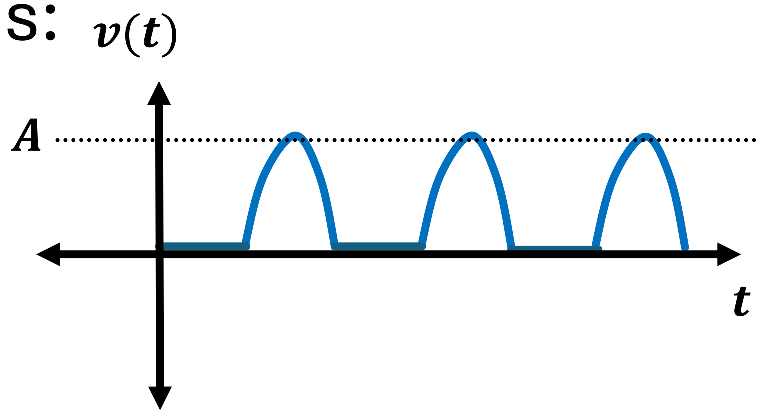
Frequency Spectrum of spike at f_o



- Not much we can do with this signal...
- But if we run that signal through a non-linear device that lets only the top half of the sinusoids through...we have something...

Fourier Theory:

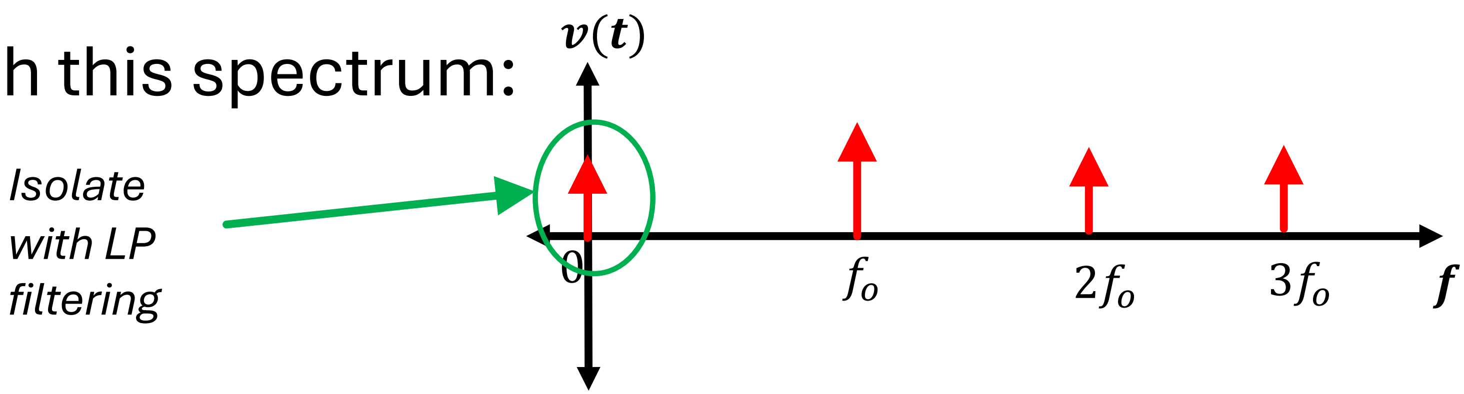
- If you rectify the signal, truncate signal's bottom half you get a signal like this:



- Which is actually comprised of a sum of sinusoids like this:

$$v_s(t) = \frac{A}{\pi} + \frac{A}{2}\sin(2\pi f_o t) - \frac{2A}{1.3}\cos(4\pi f_o t) - \frac{2A}{3.5}\cos(6\pi f_o t) + \cdots$$

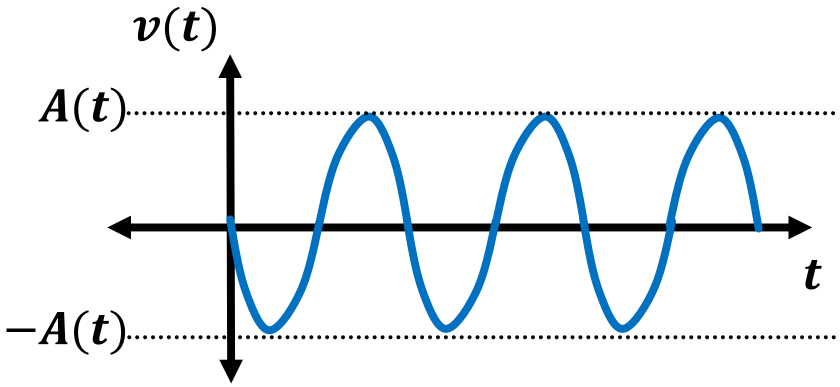
- With this spectrum:



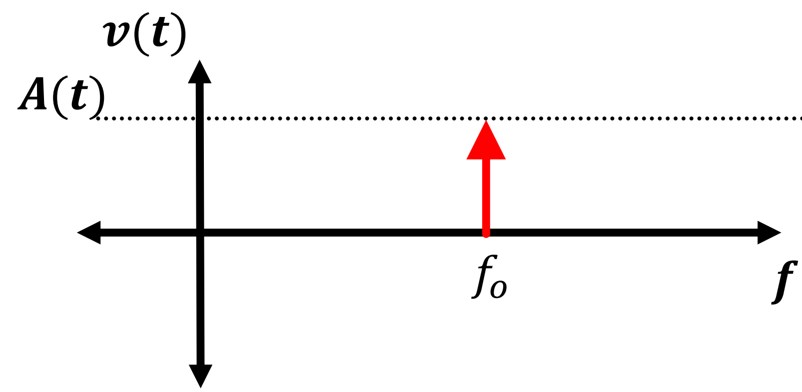
Fourier Theory Now with Time-varying A :

- If $A(t)$ could vary with time (at frequencies much lower than carrier), then the following…

Incoming signal of $v_s(t) = A(t)\sin(2\pi f_o t)$



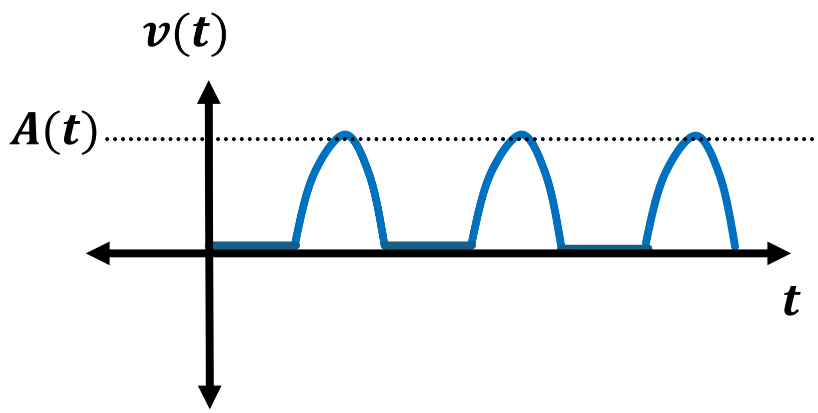
Frequency Spectrum of spike at f_o



- Run through a rectifying device:…

Fourier Theory Now with Time-varying A :

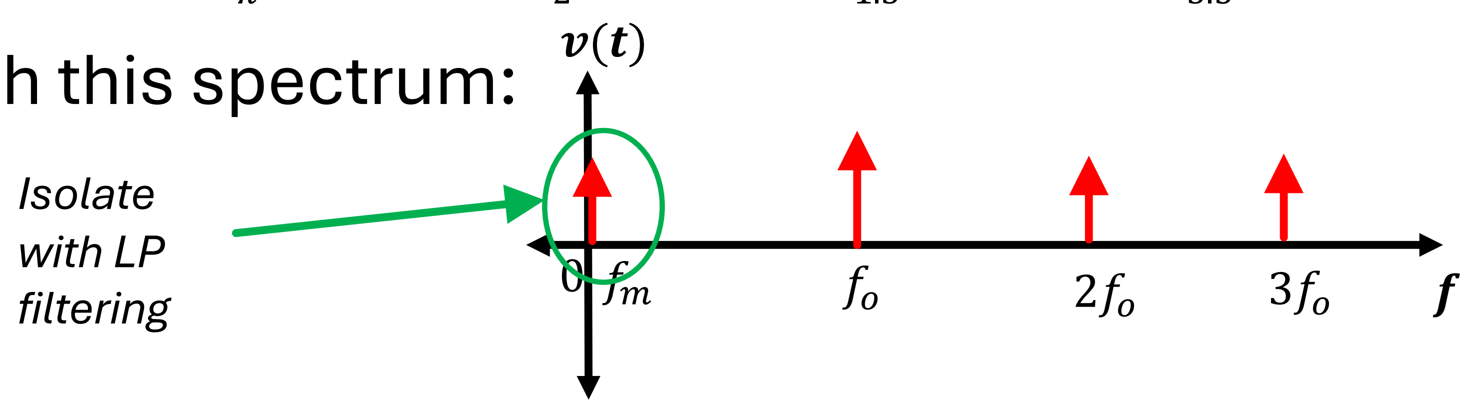
- If you truncate signal's bottom half you get a signal like this:



- Which is actually comprised of a sum of sinusoids like this:

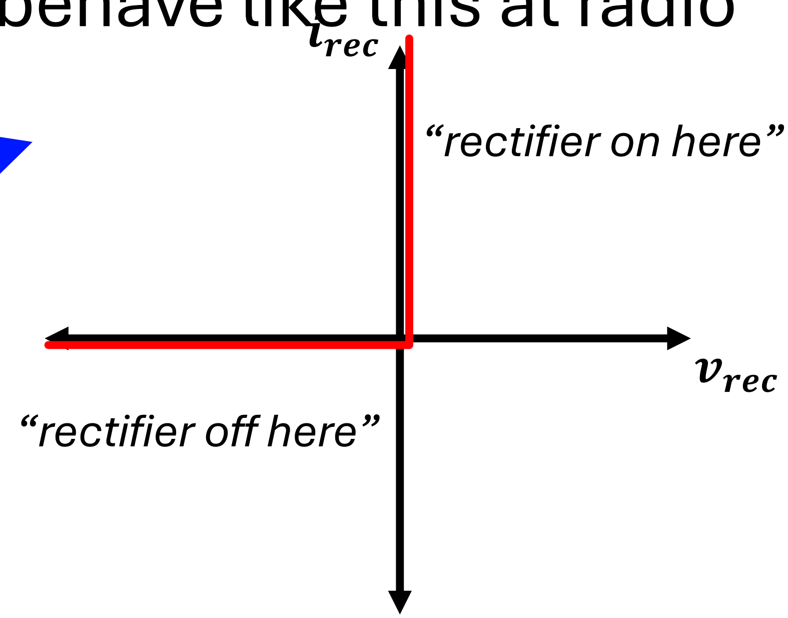
$$v_s(t) = \frac{A}{\pi}\sin(2\pi f_m t) + \frac{A}{2}\sin(2\pi f_o t) - \frac{2A}{1.3}\cos(4\pi f_o t) - \frac{2A}{3.5}\cos(6\pi f_o t) + \cdots$$

- With this spectrum:



The Search Got refined to “Rectifying Devices”

- Are there places in nature/physics where a device can conduct in one direction and not the other?
- And can these situations behave like this at radio frequencies?
- The search was on…
  - This V-I relationship:
  - This sort of resistance:



$$R_{rec}(t) = \begin{cases} 0 \text{ if } \boldsymbol{v_{rec}} > 0\text{V} \\ \infty \text{ if } \boldsymbol{v_{rec}} \leq 0\text{V} \end{cases}$$

Early Radio Detectors

- Spark Gap
- Coherers
- Electrolytic
- Magnetic
- Thin-film and Capillary
- Thermal
- Ticklers, tone-wheels
- Miscellaneous:
  - Frog-legs and brains
- Crystal Detectors

Rectifying Devices

Developments

- Since Coherers worked a lot of research was launched around the interaction of metal contacts with “things”.

- Eventually this caused a smart person to investigate how metal and semiconductors worked together…

The Winner: “Crystals”

- Through lots of experimentation and trial an error, Metal-crystal junctions started to pull ahead and work really well

- Metal was metal… nothing new there
- Crystal referred to semiconductors…
  - A lot of weird ones were used early on…
  - Lead-Sulfide (II) aka “Galena” was a popular one by early 1900s since you could basically find it ready-made in the earth*

*Many other semiconductors only appear “hidden” inside other compounds…ex: Silicon largely exists naturally only as Silicon Oxide… need good metallurgy to purify

Jagadish Bose

- Indian/Bangladeshi researcher Jagadish Bose in Calcutta who developed/refined the crystal device
- Got it working in 1901
- Didn't bother to patent it
- ”Diode” wasn't really term yet…instead the device was called a “carbon-mercury-metal coherer”

Bose

https://en.wikipedia.org/wiki/Jagadish_Chandra_Bose

Bose v. Marconi

- Marconi and his various companies were the first to successfully transmit information over the ocean.
- In 1901 he claims to have achieved first successful transmission of ‘s’ across ocean
- In order to have a detector with enough sensitivity he used an “Italian Navy Coherer”…which was actually a metal-semiconductor junction device he claimed as his own and patented.
- Marconi was very sketchy about what detectors he actually used… point of controversy ever since

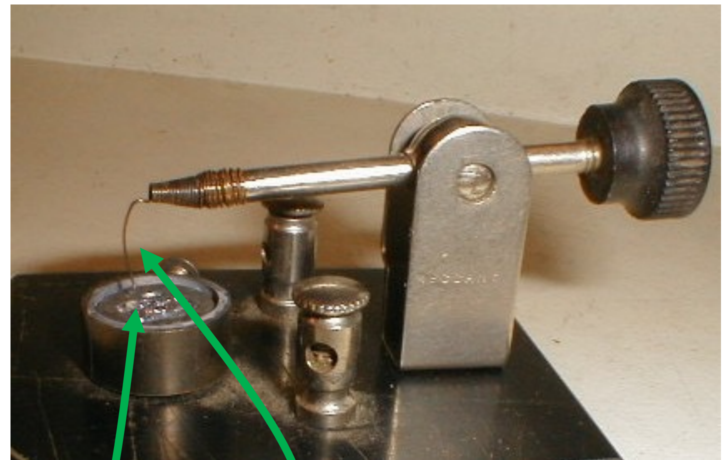
Bose

Marconi

https://en.wikipedia.org/wiki/Jagadish_Chandra_Bose

A Crystal Detector

~1905



Chunk of galena

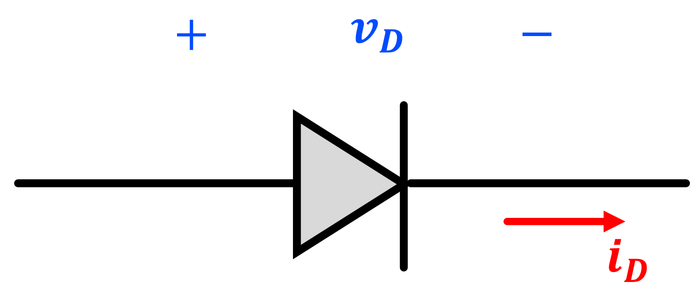
Tiny metal wire you'd poke around to get a good metal-semiconductor junction. Wire called a "Cat's Whisker"

- Traditional Coherers stuck around for Morse Code, but crystal diode detectors started to become more commonplace for better sensitivity **and for audio**

- This metal-semiconductor junction formed a point-contact diode
- Galena was not uniform…so you'd have to poke around with the whisker

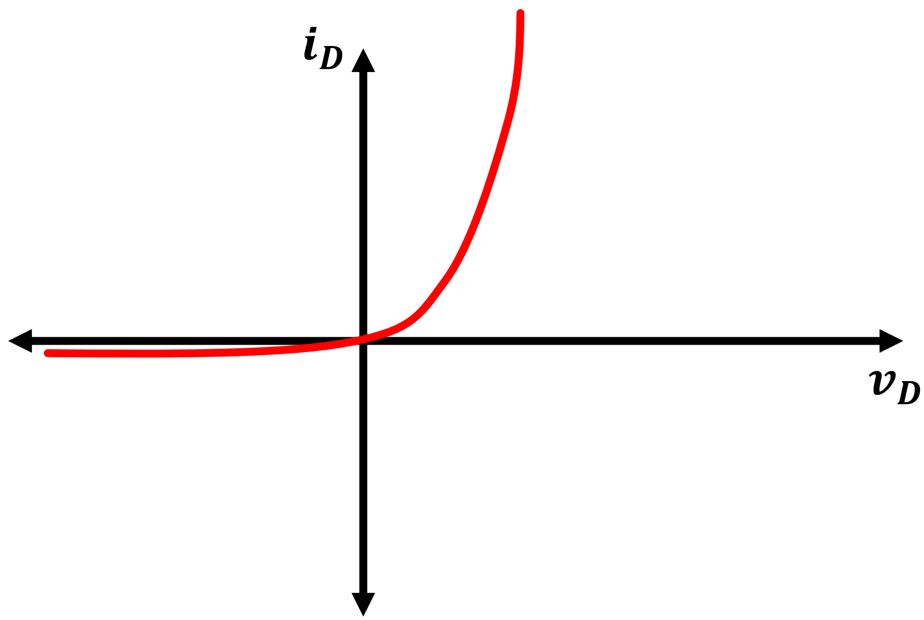
https://en.wikipedia.org/wiki/Crystal_detector

Diode

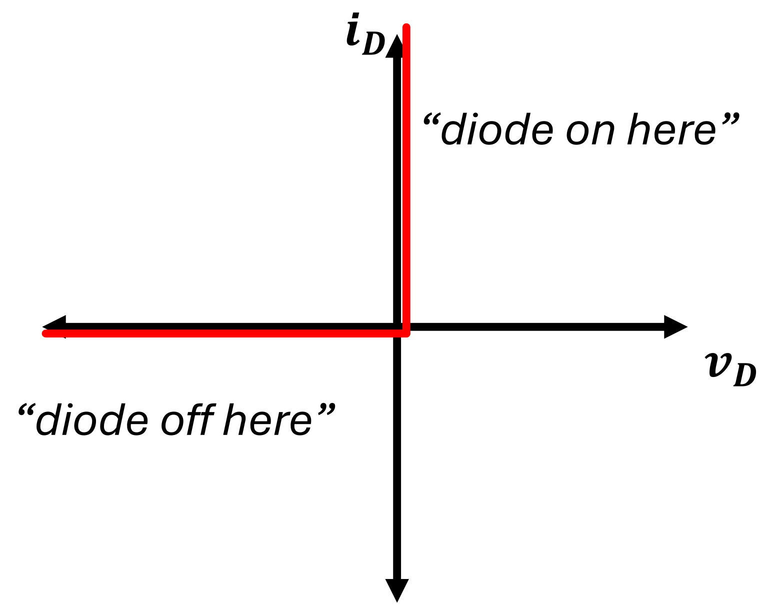


- This metal-semiconductor device is now known to us as a "diode" today. But that terminology didn't arrive until the early 1920s.

I-V characteristics looked like this:

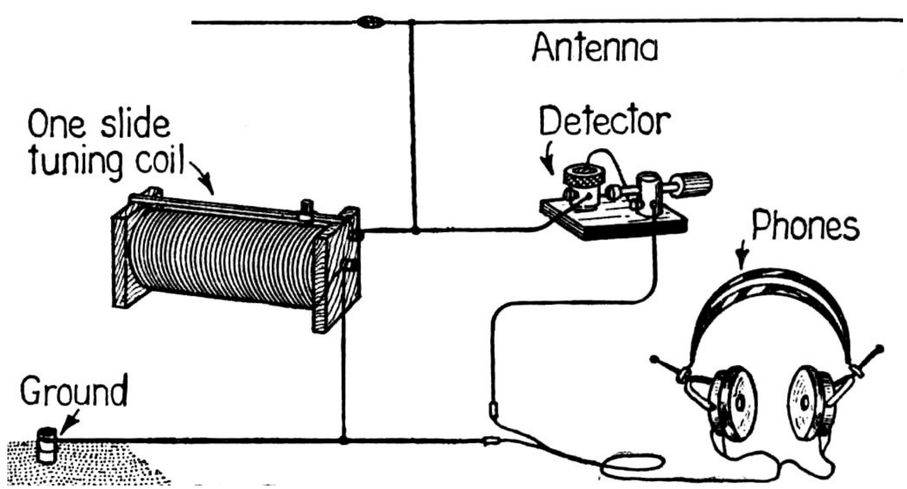


Not exactly this, but pretty close:



Crystal Radio (~1910)

- Lead Sulfide
- Make a diode from metal-semiconductor junction
- Recover radio signal
- No amplification so headsets only



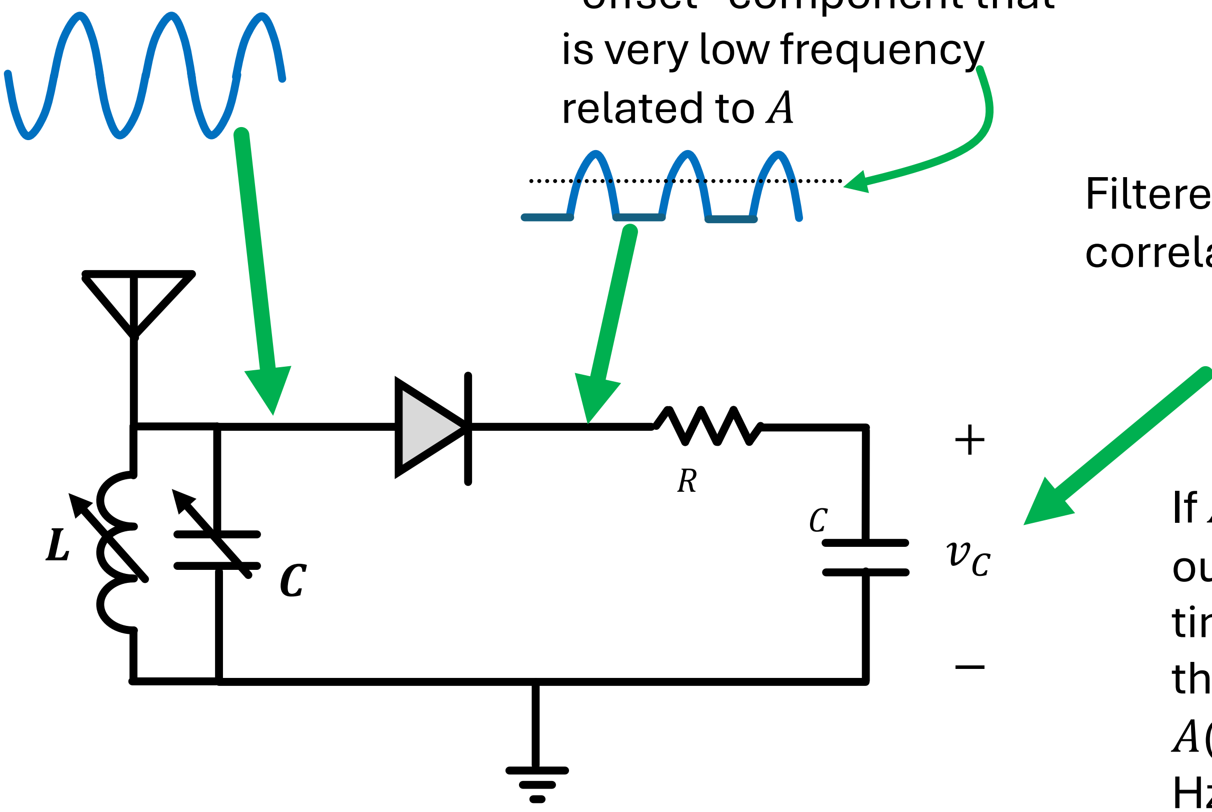
https://en.wikipedia.org/wiki/Crystal_radio#/media/File:CrystalRadio.jpg

Recovery Circuit

Our chosen frequency. Its A encodes information:

Rectified signal. This signal contains an "offset" component that is very low frequency related to A

Filtered Output is a voltage correlated with A (and only A)!

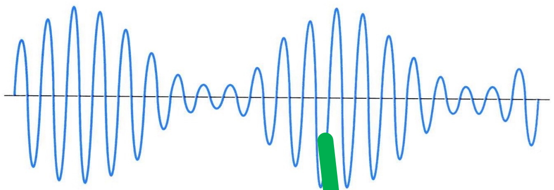


If A is actually $A(t)$, this output will track over time. All we need is for the frequency content of $A(t)$ to be much lower in Hz than the carrier frequency (which is the case even for audio!)

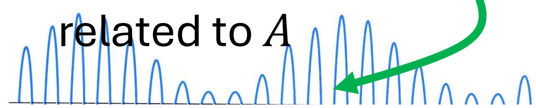
- Beautiful!

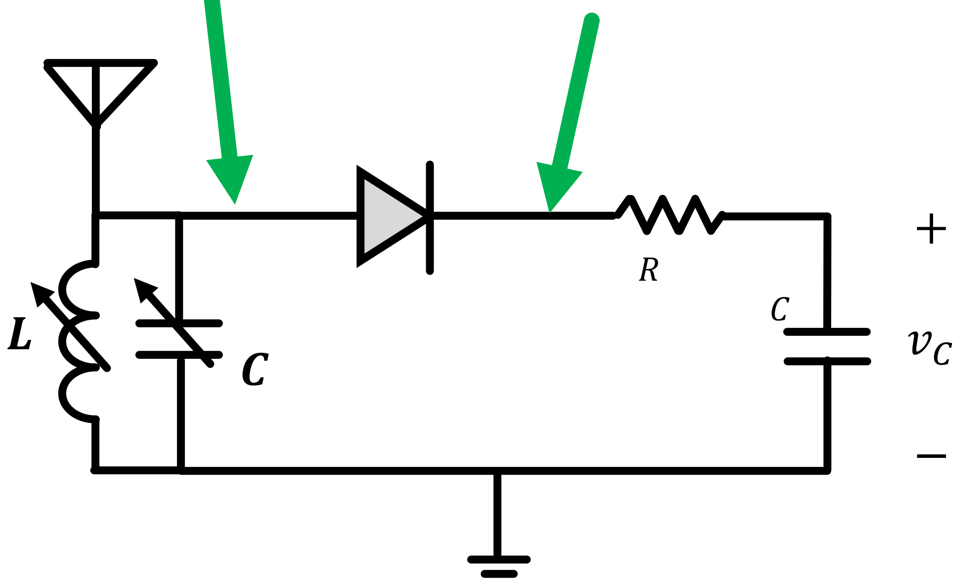
Recovery Circuit with Time-varying A

Our chosen frequency. Its A encodes information:

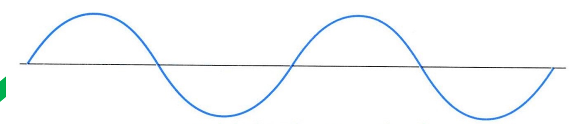


Rectified signal. This signal contains an “offset” component that is very low frequency related to A





Filtered Output is a voltage correlated with A (and only A)!



If A is actually $A(\mathrm{t})$, this output will track over time. All we need is for the frequency content of $A(\mathrm{t})$ to be much lower in Hz than the carrier frequency (which is the case even for audio!)

- Beautiful!

https://electronics.stackexchange.com/questions/296616/am-demodulation-with-envelope-detector

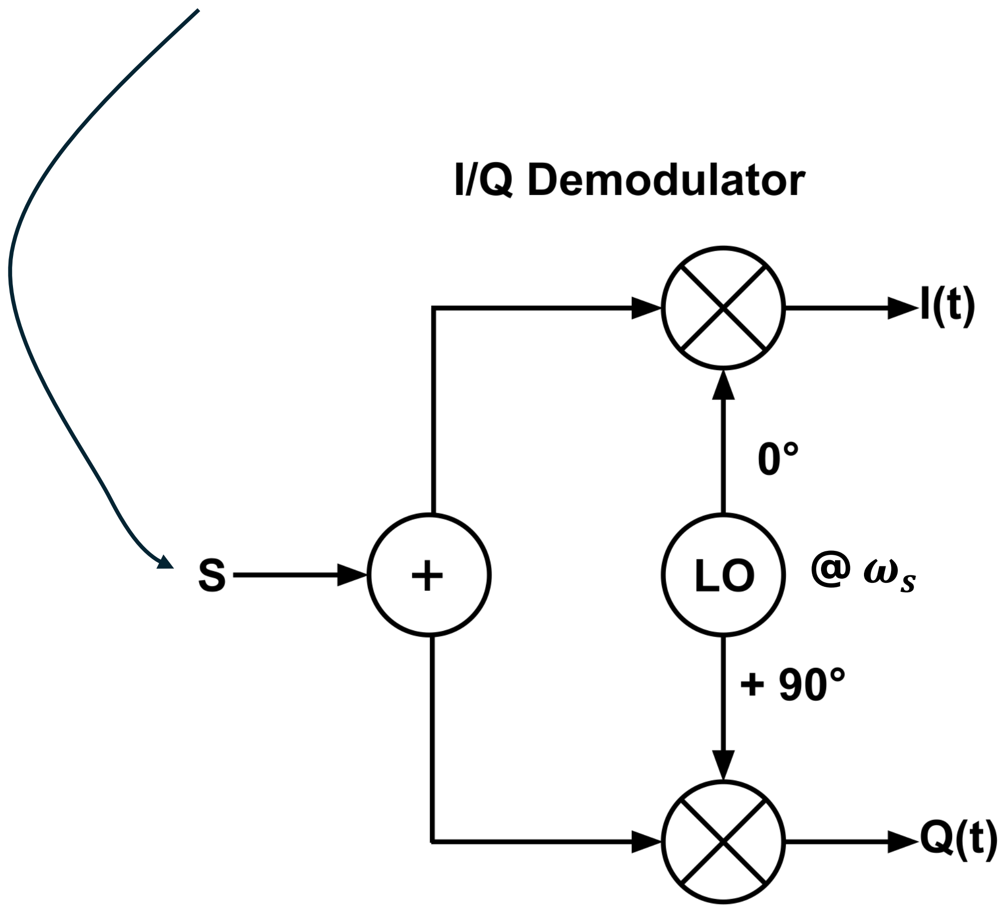
Return to situation

$$v(t) = A(t)\cos(\omega_s t + \phi(t))$$

- Let's say you want to measure an incoming sine wave with some fixed known frequency ω_s and you want to determine what the amplitude and phase are

- How would you do that?

$$v(t) = A(t)\cos(\omega_s t + \phi(t))$$



https://en.wikipedia.org/wiki/In-phase_and_quadrature_components#/media/File:IQ_Mod_Demod_block_2.svg

$$s(t) = A_m \cos(\omega_c t + \phi_m)$$

$$\frac{A_m e^{j(\omega_c t+\phi_m)}}{2} + \frac{A_m e^{-j(\omega_c t+\phi_m)}}{2}$$

$$\frac{A_m}{2}\cos(\omega_c t + \phi_m) + j\sin(\omega_c t + \phi_m) \qquad \frac{A_m}{2}\cos(\omega_c t + \phi_m) - j\sin(\omega_c t + \phi_m)$$

$$\left(\frac{A_m}{2}\cos(\omega_c t) + j\frac{A_m}{2}\sin(\omega_c t)\right)(\cos(\phi_m) + j\sin(\phi_m)) \;+\; \left(\frac{A_m}{2}\cos(\omega_c t) - j\frac{A_m}{2}\sin(\omega_c t)\right)(\cos(\phi_m) - j\sin(\phi_m))$$

$$\frac{A_m}{2}\cos(\omega_c t)\cos(\phi_m) + j\frac{A_m}{2}\cos(\omega_c t)\sin(\phi_m) + j\frac{A_m}{2}\sin(\omega_c t)\cos(\phi_m) - \frac{A_m}{2}\sin(\omega_c t)\sin(\phi_m) +$$
$$\frac{A_m}{2}\cos(\omega_c t)\cos(\phi_m) - j\frac{A_m}{2}\cos(\omega_c t)\sin(\phi_m) - j\frac{A_m}{2}\sin(\omega_c t)\cos(\phi_m) - \frac{A_m}{2}\sin(\omega_c t)\sin(\phi_m)$$

$$A_m \cos(\phi_m)\cos(\omega_c t) - A_m \sin(\phi_m)\sin(\omega_c t)$$

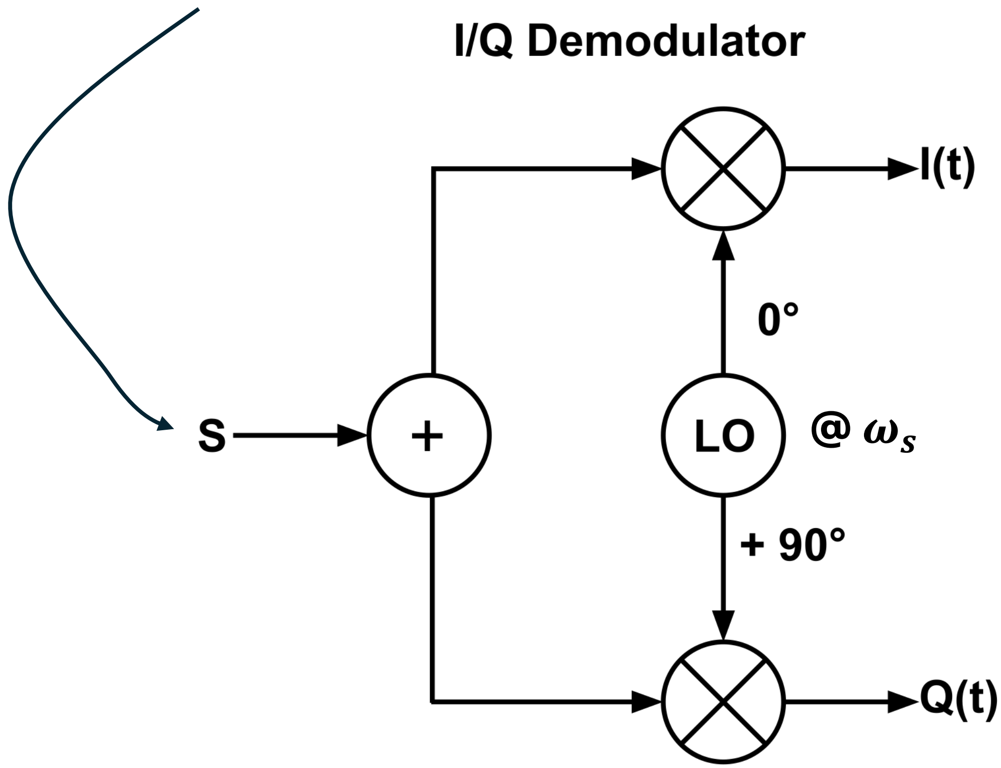
Any Cos/sine wave that is running…

$$A(t)\cos\big(\omega_c t + \phi(t)\big) =$$

$$= \underbrace{\cos(\omega_c t)\cdot A(t)\cos(\phi(t))}_{\textit{In-phase component}} - \underbrace{\sin(\omega_c t)\cdot A(t)\sin(\phi(t))}_{\textit{Quadrature component}}$$

Quadrature Sampling…How Does it Help?

$$v(t) = A(t)\cos(\omega_s t + \phi(t))$$



https://en.wikipedia.org/wiki/In-phase_and_quadrature_components#/media/File:IQ_Mod_Demod_block_2.svg

To harvest the information from that modulated wave the old way…

- Capture the wave from its medium:

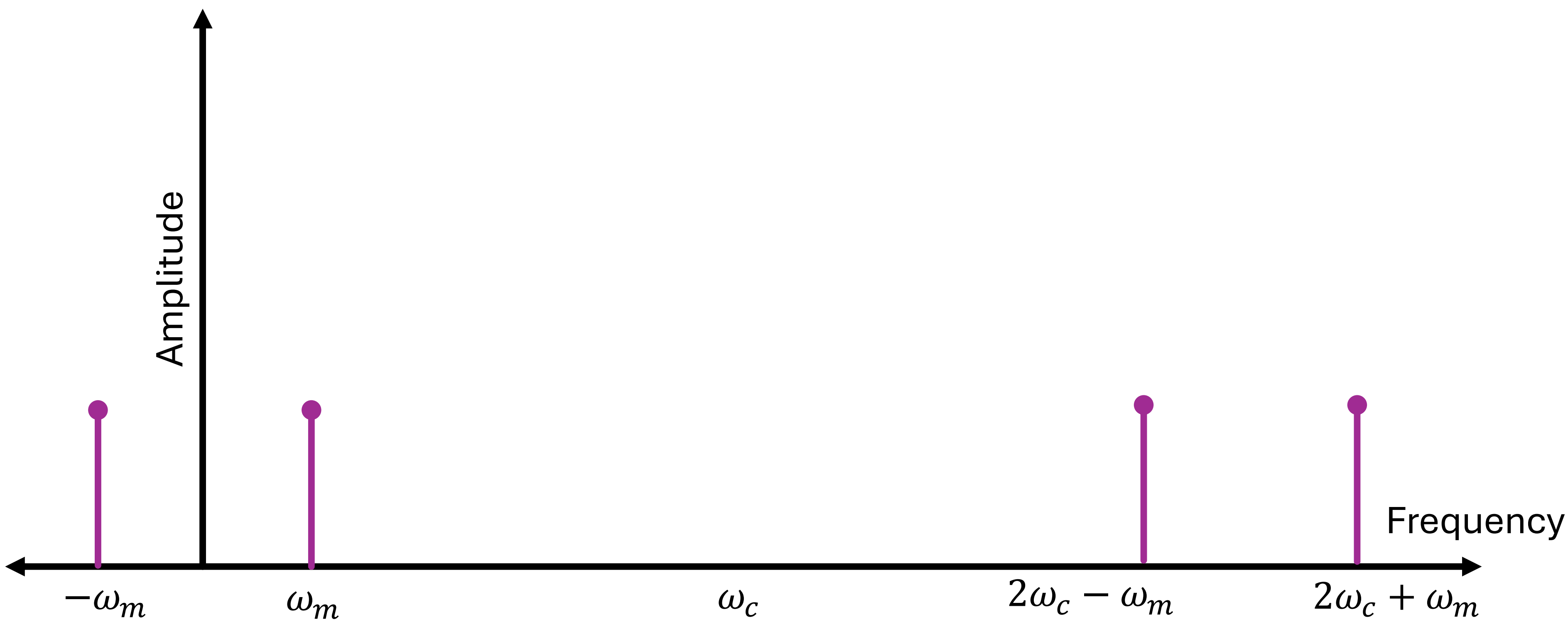
$$v(t) = \frac{A}{2} \cdot \cos\big((\omega_c + \omega_m)t\big) + \frac{A}{2} \cdot \cos\big((\omega_c - \omega_m)t\big)$$

- Multiply it again by the targeted carrier wave $\cos(\omega_c t)$ (just like before to get four terms…)

$$v(t) = \frac{A}{4} \cdot \cos\big((\omega_c + \omega_m + \omega_c)t\big) + \frac{A}{4} \cdot \cos\big((\omega_c + \omega_m - \omega_c)t\big)$$
$$+ \frac{A}{4} \cdot \cos\big((\omega_c - \omega_m + \omega_c)t\big) + \frac{A}{4} \cdot \cos\big((\omega_c - \omega_m - \omega_c)t\big)$$

And then… You get four different terms…

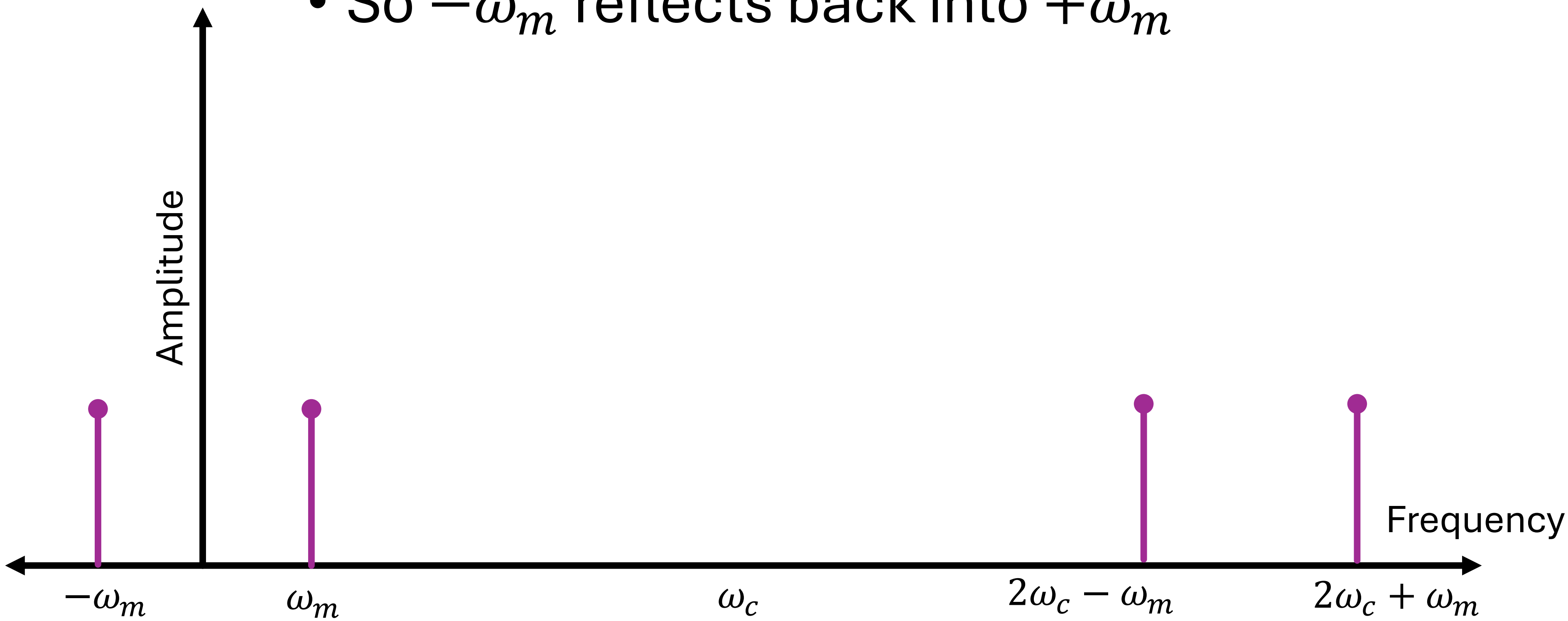
$$v(t) = \frac{A}{4} \cdot \cos\big((\omega_c + \omega_m + \omega_c)t\big) + \frac{A}{4} \cdot \cos\big((\omega_c + \omega_m - \omega_c)t\big)$$
$$+ \frac{A}{4} \cdot \cos\big((\omega_c - \omega_m + \omega_c)t\big) + \frac{A}{4} \cdot \cos\big((\omega_c - \omega_m - \omega_c)t\big)$$



Cos(-x) = cos(x)

$$v(t) = \frac{A}{4} \cdot \cos\big((\omega_c + \omega_m + \omega_c)t\big) + \frac{A}{4} \cdot \cos\big((\omega_c + \omega_m - \omega_c)t\big)$$
$$+ \frac{A}{4} \cdot \cos\big((\omega_c - \omega_m + \omega_c)t\big) + \frac{A}{4} \cdot \cos\big((\omega_c - \omega_m - \omega_c)t\big)$$

- So $-\omega_m$ reflects back into $+\omega_m$



LPF away that extra trash up high…

$$v(t) = \frac{A}{4} \cdot \cos\big((\omega_c + \omega_m + \omega_c)t\big) + \frac{A}{4} \cdot \cos\big((\omega_c + \omega_m - \omega_c)t\big)$$
$$+ \frac{A}{4} \cdot \cos\big((\omega_c - \omega_m + \omega_c)t\big) + \frac{A}{4} \cdot \cos\big((\omega_c - \omega_m - \omega_c)t\big)$$

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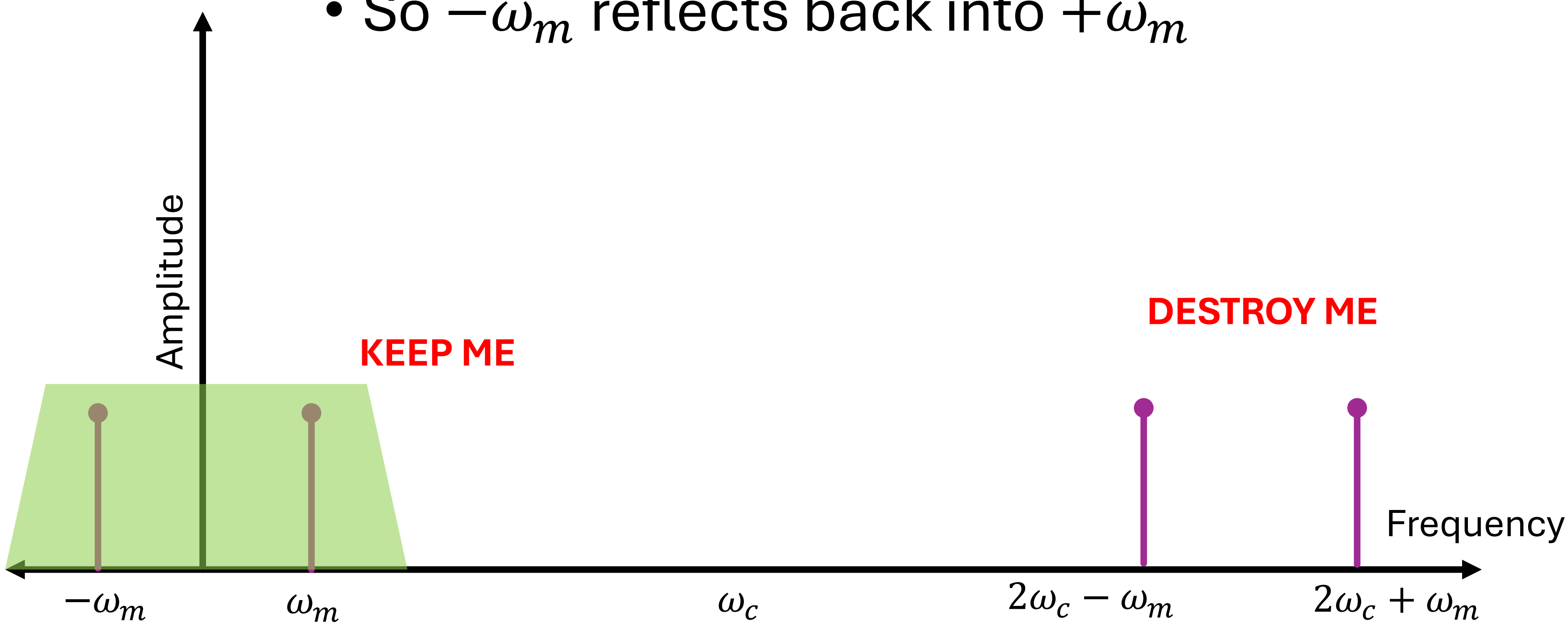


The fact that we transmit redundant information doesn't really matter…

- Since we can't really "see" it as negative anyways.

- But what if we're mixing it with a sine wave?

Trig Identities

Product-to-sum identities [edit]

The product of two sines or cosines of different angles can be converted to a sum of trigonometric functions of a sum and difference of those angles:

$$\cos\theta\,\cos\varphi = \tfrac{1}{2}\big(\cos(\theta-\varphi)+\cos(\theta+\varphi)\big),$$
$$\sin\theta\,\sin\varphi = \tfrac{1}{2}\big(\cos(\theta-\varphi)-\cos(\theta+\varphi)\big),$$
$$\sin\theta\,\cos\varphi = \tfrac{1}{2}\big(\sin(\theta+\varphi)+\sin(\theta-\varphi)\big),$$
$$\cos\theta\,\sin\varphi = \tfrac{1}{2}\big(\sin(\theta+\varphi)-\sin(\theta-\varphi)\big).$$

- cos × cos leads to just cos
- But cos × sin leads to some sin terms and those do not have even symmetry across 0

What if we mixed with a Sine Wave?

- Capture the wave from its medium:

$$v(t) = \frac{A}{2} \cdot \cos\big((\omega_c + \omega_m)t\big) + \frac{A}{2} \cdot \cos\big((\omega_c - \omega_m)t\big)$$

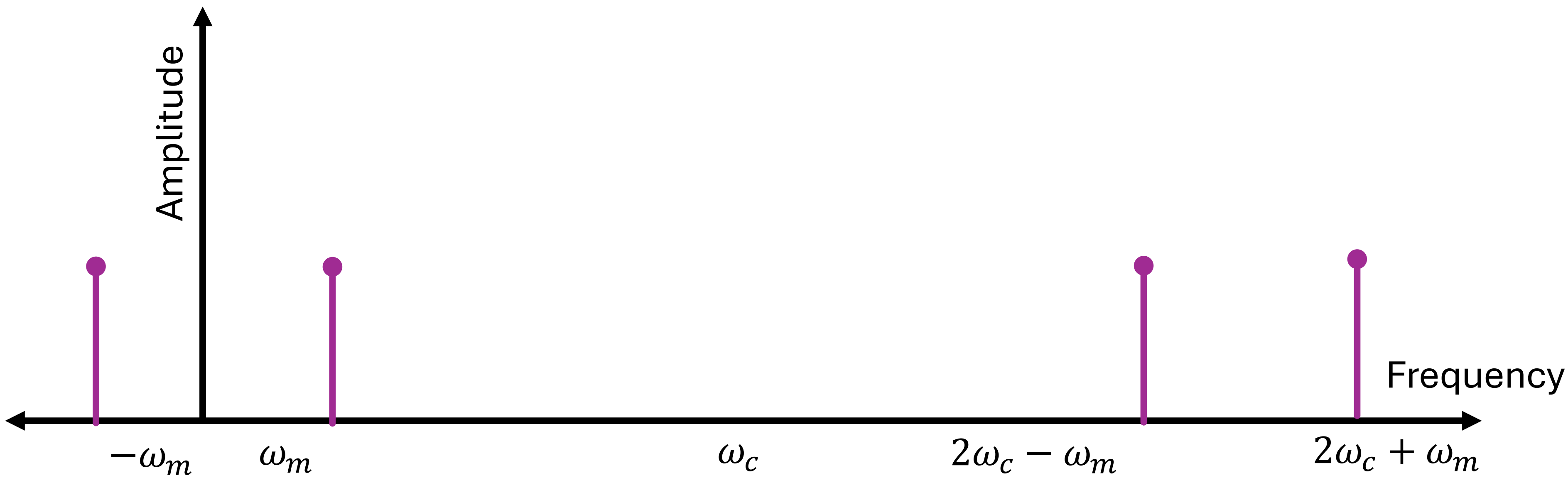
- Multiply it again by the targeted carrier wave $\mathbf{sin}(\boldsymbol{\omega_c t})$ (just like before to get four terms…)

$$v(t) = \frac{A}{4} \cdot \sin\big((\omega_c + \omega_m + \omega_c)t\big) - \frac{A}{4} \cdot \sin\big((\omega_c + \omega_m - \omega_c)t\big)$$
$$+ \frac{A}{4} \cdot \sin\big((\omega_c - \omega_m + \omega_c)t\big) - \frac{A}{4} \cdot \sin\big((\omega_c - \omega_m - \omega_c)t\big)$$

And then... You get four different terms...

$$v(t) = \frac{A}{4} \cdot \sin\big((\omega_c + \omega_m + \omega_c)t\big) - \frac{A}{4} \cdot \sin\big((\omega_c + \omega_m - \omega_c)t\big)$$
$$+ \frac{A}{4} \cdot \sin\big((\omega_c - \omega_m + \omega_c)t\big) - \frac{A}{4} \cdot \sin\big((\omega_c - \omega_m - \omega_c)t\big)$$

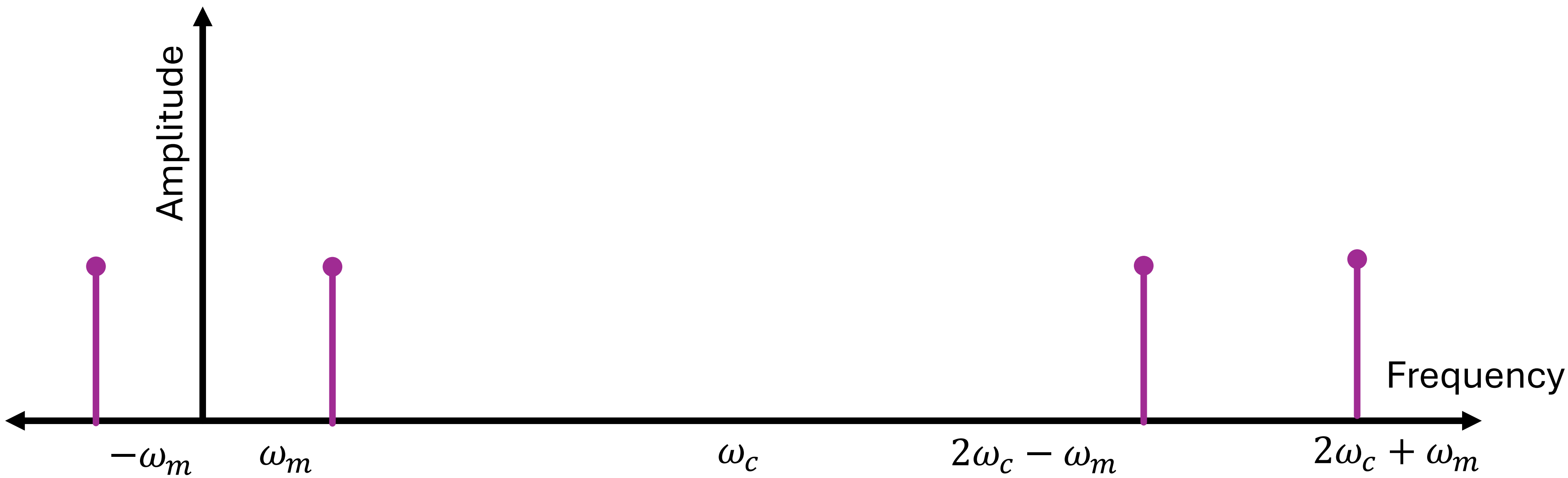
Remember these are amplitudes!



Sin(-x) != sin(x)....is different

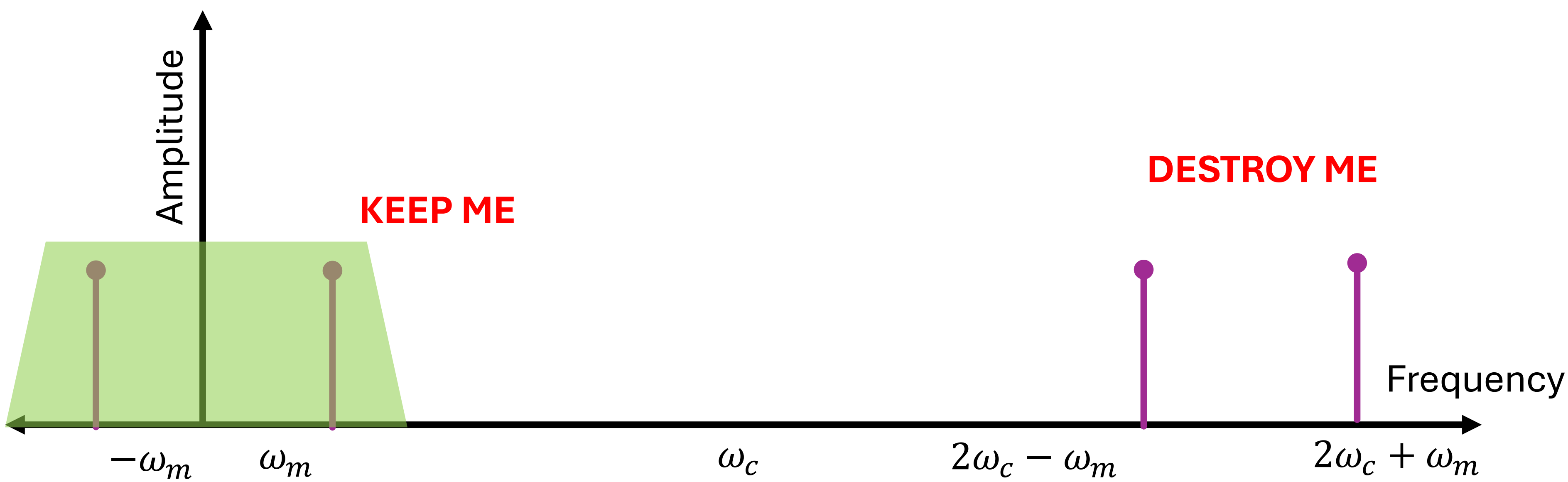
$$v(t) = \frac{A}{4} \cdot \sin\big((\omega_c + \omega_m + \omega_c)t\big) - \frac{A}{4} \cdot \sin\big((\omega_c + \omega_m - \omega_c)t\big)$$
$$+ \frac{A}{4} \cdot \sin\big((\omega_c - \omega_m + \omega_c)t\big) - \frac{A}{4} \cdot \sin\big((\omega_c - \omega_m - \omega_c)t\big)$$

Remember these are amplitudes!



LPF away that extra trash up high…

$$v(t) = \frac{A}{4} \cdot \sin\big((\omega_c + \omega_m + \omega_c)t\big) - \frac{A}{4} \cdot \sin\big((\omega_c + \omega_m - \omega_c)t\big)$$
$$+ \frac{A}{4} \cdot \sin\big((\omega_c - \omega_m + \omega_c)t\big) - \frac{A}{4} \cdot \sin\big((\omega_c - \omega_m - \omega_c)t\big)$$



Putting the results together.

- Separately the results of each of these mixings after filtering aren't that amazing…

$$v_{cos}(t) = \frac{A}{4} \cdot \cos\left((\omega_m)t\right) + \frac{A}{4} \cdot \cos\left((-\omega_m)t\right)$$

$$v_{sin}\,(t) = -\frac{A}{4} \cdot \sin\left((\omega_m)t\right) - \frac{A}{4} \cdot \sin\left((-\omega_m)t\right)$$

- Grouping half of each pair of results together though

Two Signals Show Up!

$$v_{s1}(t) = \frac{A}{4} \cdot \cos\big((\omega_m)t\big) - \frac{A}{4} \cdot \sin\big((\omega_m)t\big)$$

$$v_{s2}(t) = \frac{A}{4} \cdot \cos\big((-\omega_m)t\big) - \frac{A}{4} \cdot \sin\big((-\omega_m)t\big)$$

Note to Joe: go to sweet Desmos animation

What’s also important

- The signal at $+\omega_m$ and $-\omega_m$ is now fully distinguishable the whole time through this pipeline…they never get mixed/reflected into one another.

- This means the waste of the double-sideband that comes from regular mixing may actually not be ignorable anymore…there’s actually a way to distinguish upper and lower sideband content

What about just using Sine?

- That seems to give us the difference on its own since sin(-x) != sin(x)

- Problem is we don't really know if we have sine ever because of phase offsets that add an additional unknown

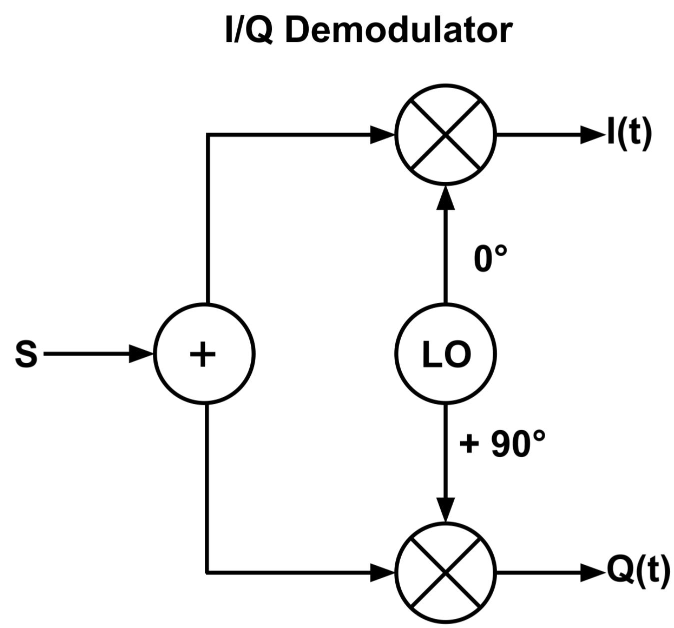
- Having the two signals in quadrature allows us to always have a fixed 90 degree and things cancel out in the end.

$$s(t) = A_m \cos(\omega_c t + \phi_m)$$

$$d_i(t) = \cos(\omega_c t + \phi_?)$$

$$d_q(t) = \sin(\omega_c t + \phi_?)$$

- There's usually one additional unknown (phase offset) we have to deal with…if we



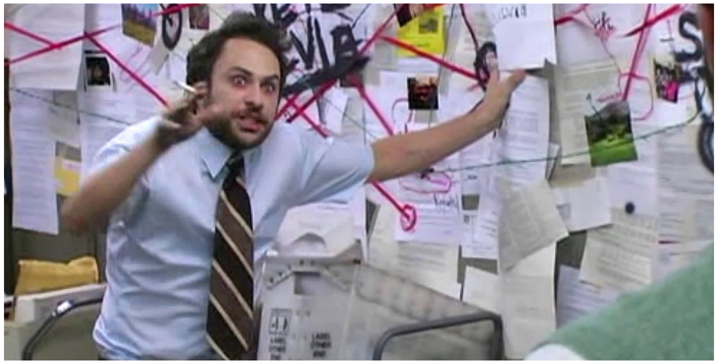
Call each of the components

- Sinusoids can therefore be thought of as sums of complex oscillating signals

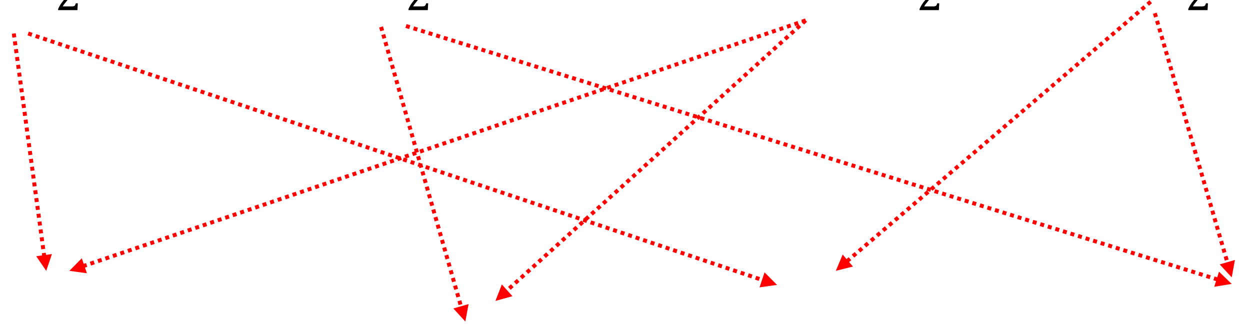
$$\cos(\alpha) = \frac{e^{j\alpha}}{2} + \frac{e^{-j\alpha}}{2}$$

$$\sin(\alpha) = \frac{je^{-j\alpha}}{2} - \frac{je^{j\alpha}}{2}$$

Just the Cosine Term Getting Mixed



$$\frac{A_m e^{j(\omega_c t+\phi_m)}}{2} + \frac{A_m e^{-j(\omega_c t+\phi_m)}}{2} \quad \times \quad \frac{e^{j(\omega_c t+\phi_?)}}{2} + \frac{e^{-j(\omega_c t+\phi_?)}}{2}$$



$$\frac{A_m e^{j(2\omega_c t+\phi_m+\phi_?)}}{4} + \frac{A_m e^{-j(\phi_m-\phi_?)}}{4} + \frac{A_m e^{j(\phi_m-\phi_?)}}{4} + \frac{A_m e^{-j(2\omega_c t+\phi_m+\phi_?)}}{4}$$

High frequency time-varying (2X carrier)…filter out

Just the Sine Term Getting Mixed

$$\frac{A_m e^{j(\omega_c t+\phi_m)}}{2}+\frac{A_m e^{-j(\omega_c t+\phi_m)}}{2} \times \frac{je^{-j(\omega_c t+\phi_?)}}{2}-\frac{je^{j(\omega_c t+\phi_?)}}{2}$$

$$\frac{A_m j e^{j(\phi_m-\phi_?)}}{4}+\frac{A_m j e^{-j(2\omega_c t+\phi_m+\phi_?)}}{4}-\frac{A_m j e^{-j(2\omega_c t+\phi_m+\phi_?)}}{4}-\frac{A_m j e^{-j(\phi_m-\phi_?)}}{4}$$

High frequency time-varying (2X carrier)…filter out

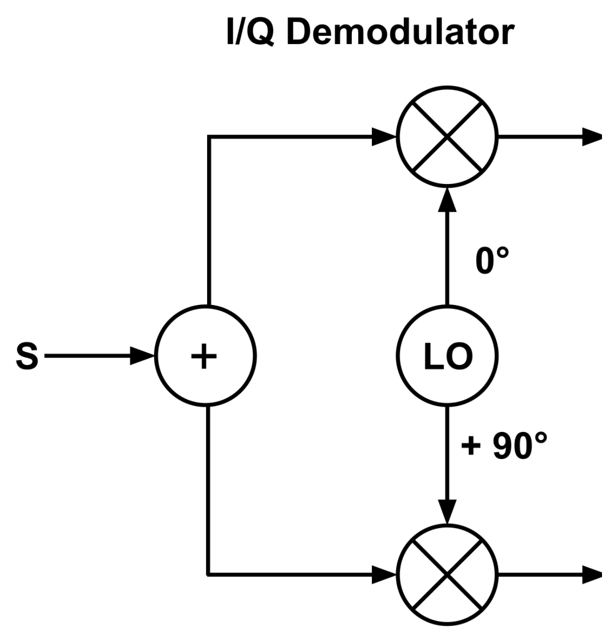
Leaves us with:

$$s(t) = A_m \cos(\omega_c t + \phi_m)$$

$$d_i(t) = \cos(\omega_c t + \phi_?)$$
$$d_q(t) = \sin(\omega_c t + \phi_?)$$

$$\frac{A_m e^{-j(\phi_m - \phi_?)}}{4} + \frac{A_m e^{j(\phi_m - \phi_?)}}{4} \qquad \frac{A_m j e^{j(\phi_m - \phi_?)}}{4} - \frac{A_m j e^{-j(\phi_m - \phi_?)}}{4}$$

$$\frac{A_m}{4} \cdot \cos(\phi_m - \phi_?) - \frac{A_m}{4} \cdot \sin(\phi_m - \phi_?)$$



So what does I/Q give us?

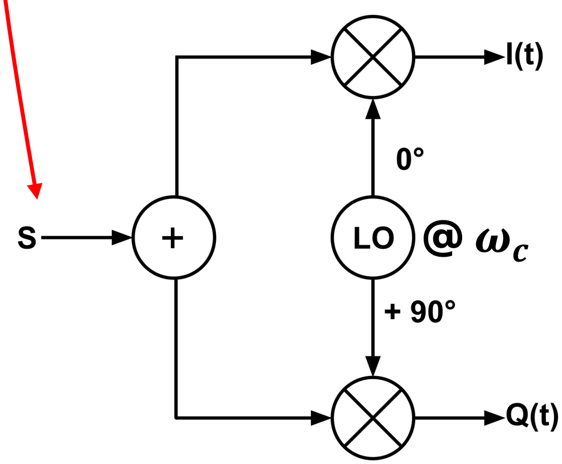
$$A(t)\cos\big(\omega_c t + \phi(t)\big) =$$

$$= \cos(\omega_c t) \cdot \underbrace{A(t)\cos(\phi(t))}_{\text{In-phase component}} - \sin(\omega_c t) \cdot \underbrace{A(t)\sin(\phi(t))}_{\text{Quadrature component}}$$

In-phase component

Quadrature component

I/Q Demodulator

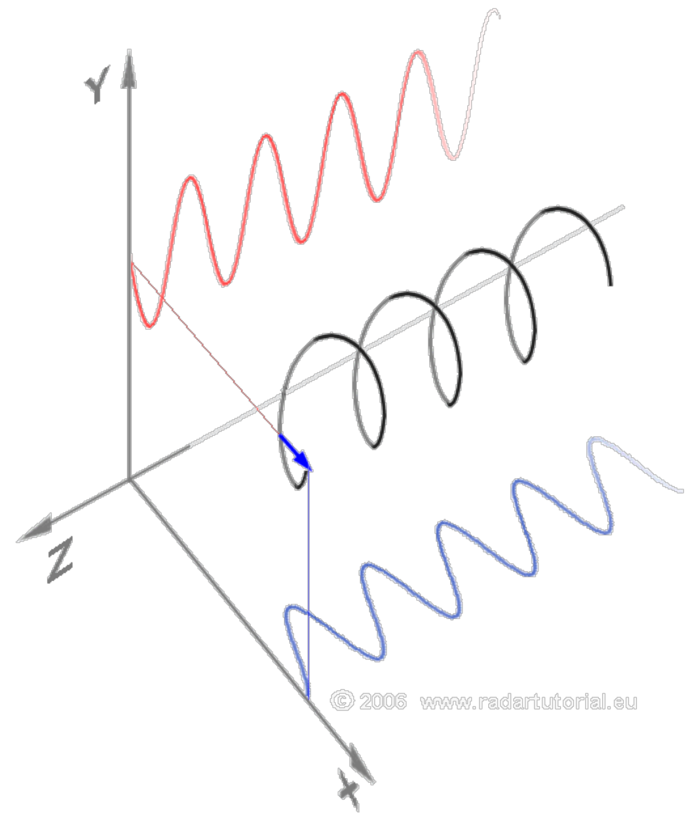


Every sample point now…

- Is a two-value tuple of I/Q
- These are enough to describe the instantaneous amplitude and phase of a signal.
- They are also completely separated from frequency content

Another Way to Think About it

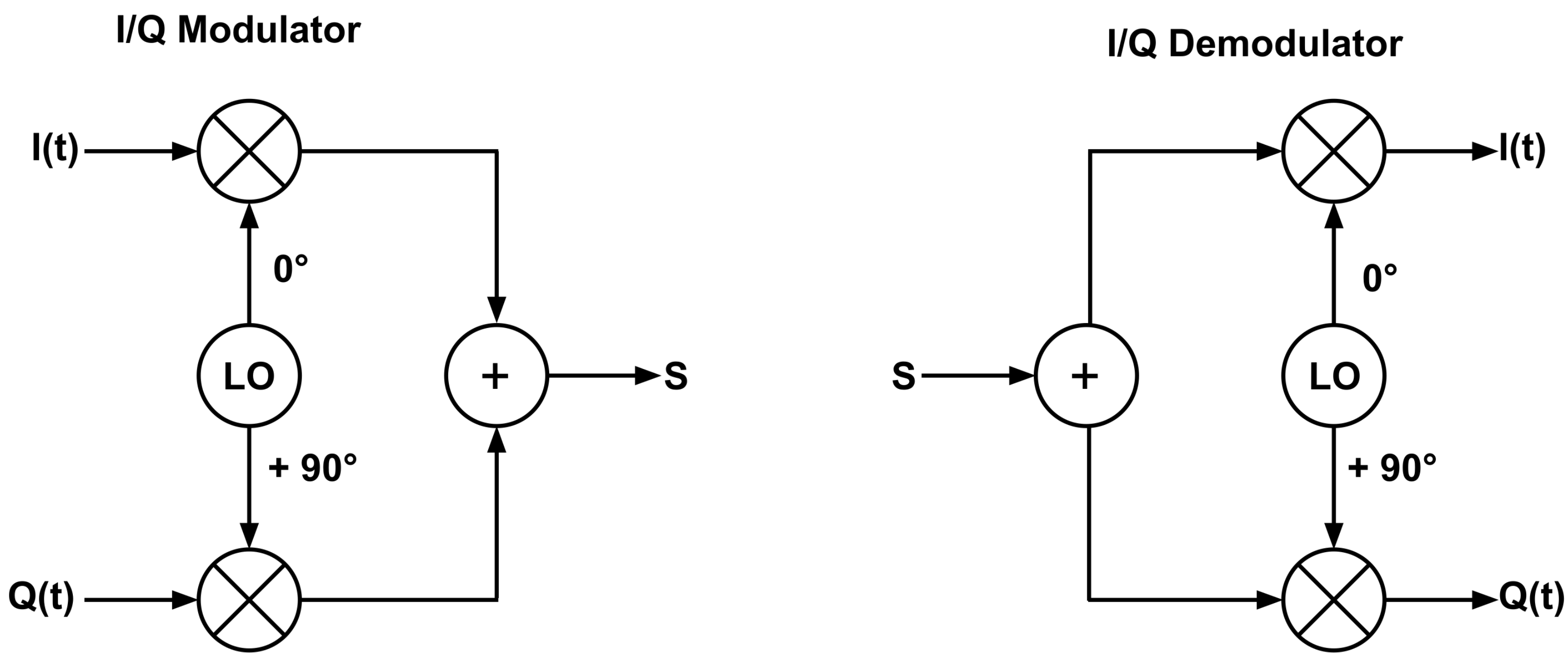
- By Measuring I/Q values we can also distinguish between positive and negative frequencies which only make sense in two-dimensions
- The I/Q are basically giving us indirect information about the 2D oscillations



What About on the Modulation Side?

I/Q Sampling Allows us to Demodulate signals…Does it help us?

- Yes!!



https://en.wikipedia.org/wiki/In-phase_and_quadrature_components#/media/File:IQ_Mod_Demod_block_2.svg

Any Cos/sine wave that is running…

$$A(t)\cos\big(\omega_c t + \phi(t)\big) =$$

$$= \underbrace{\cos(\omega_c t)\cdot A(t)\cos(\phi(t))}_{\textit{In-phase component}} - \underbrace{\sin(\omega_c t)\cdot A(t)\sin(\phi(t))}_{\textit{Quadrature component}}$$

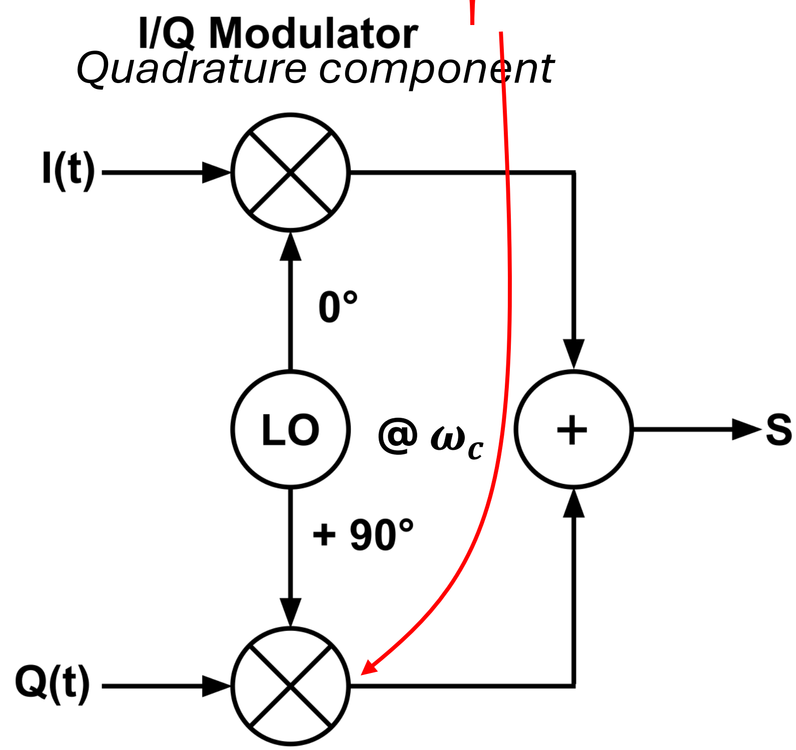
How Modulate?

$$A(t)\cos\big(\omega_c t + \phi(t)\big) =$$

$$= \cos(\omega_c t)\cdot \underbrace{A(t)\cos(\phi(t))}_{\textit{In-phase component}} - \sin(\omega_c t)\cdot \underbrace{A(t)\sin(\phi(t))}_{\textit{Quadrature component}}$$

In-phase component

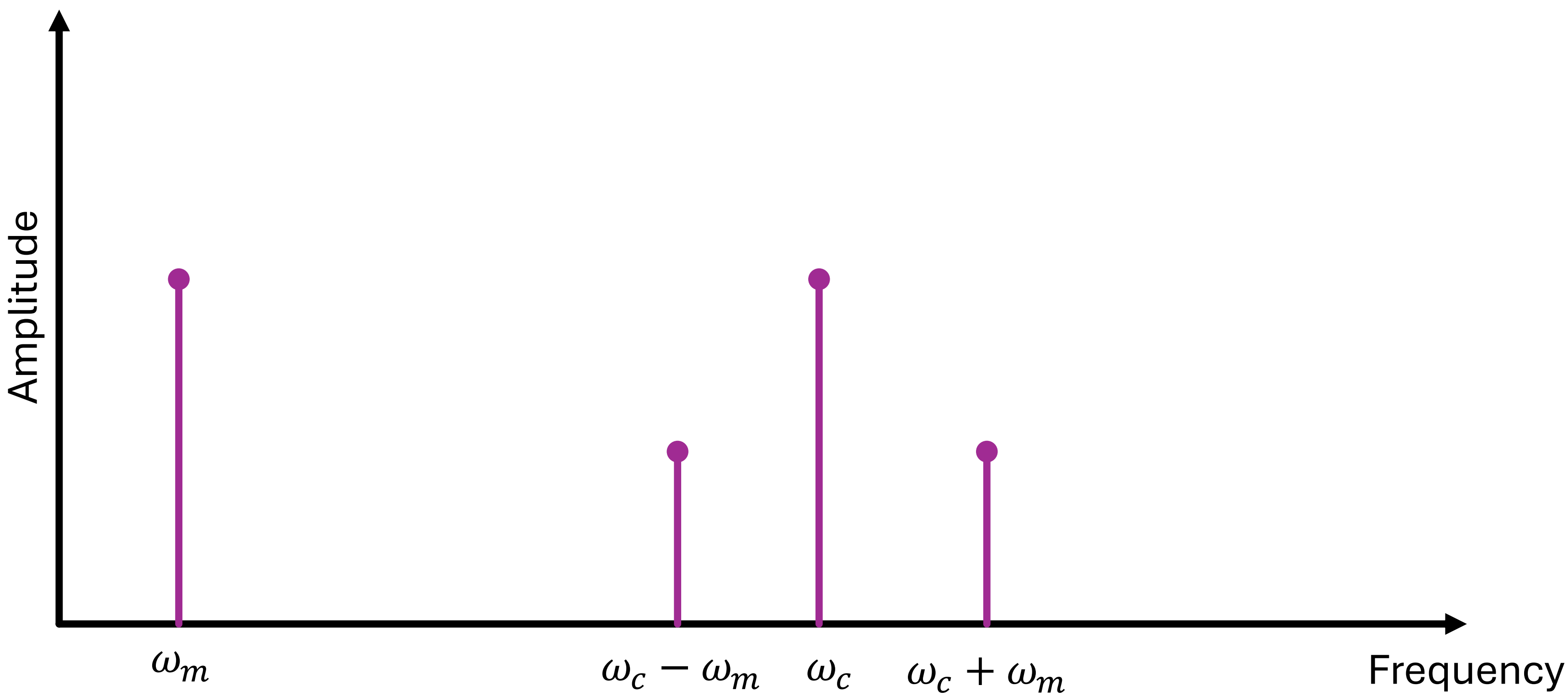
Quadrature component



- Instead of measuring 2D values, provide 2D values to two offset oscillators and combine signals
- Basically two in parallel amplitude modulations

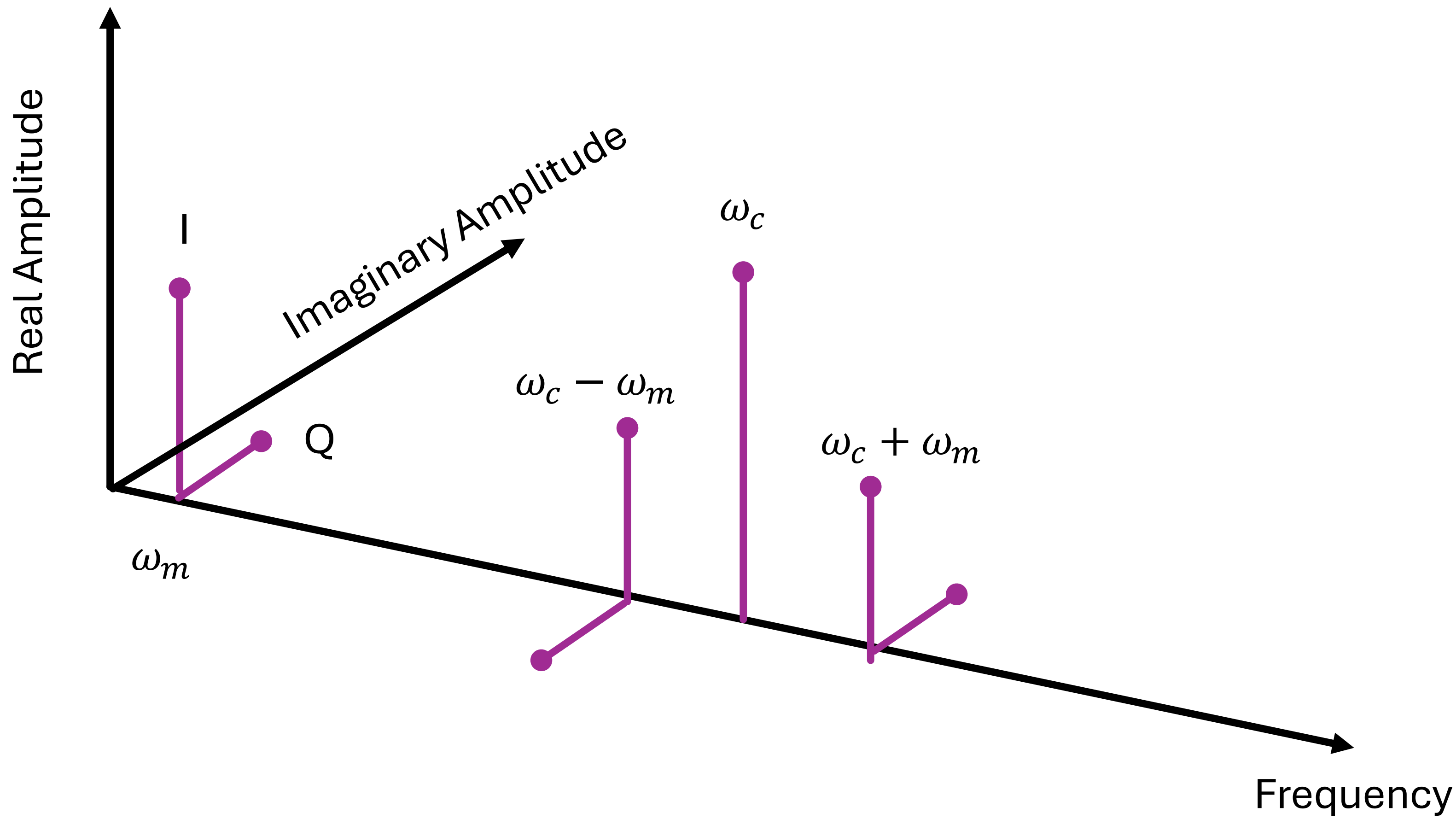
Interesting observation

- To AM one frequency, you end up generating *two frequencies…good or bad?*



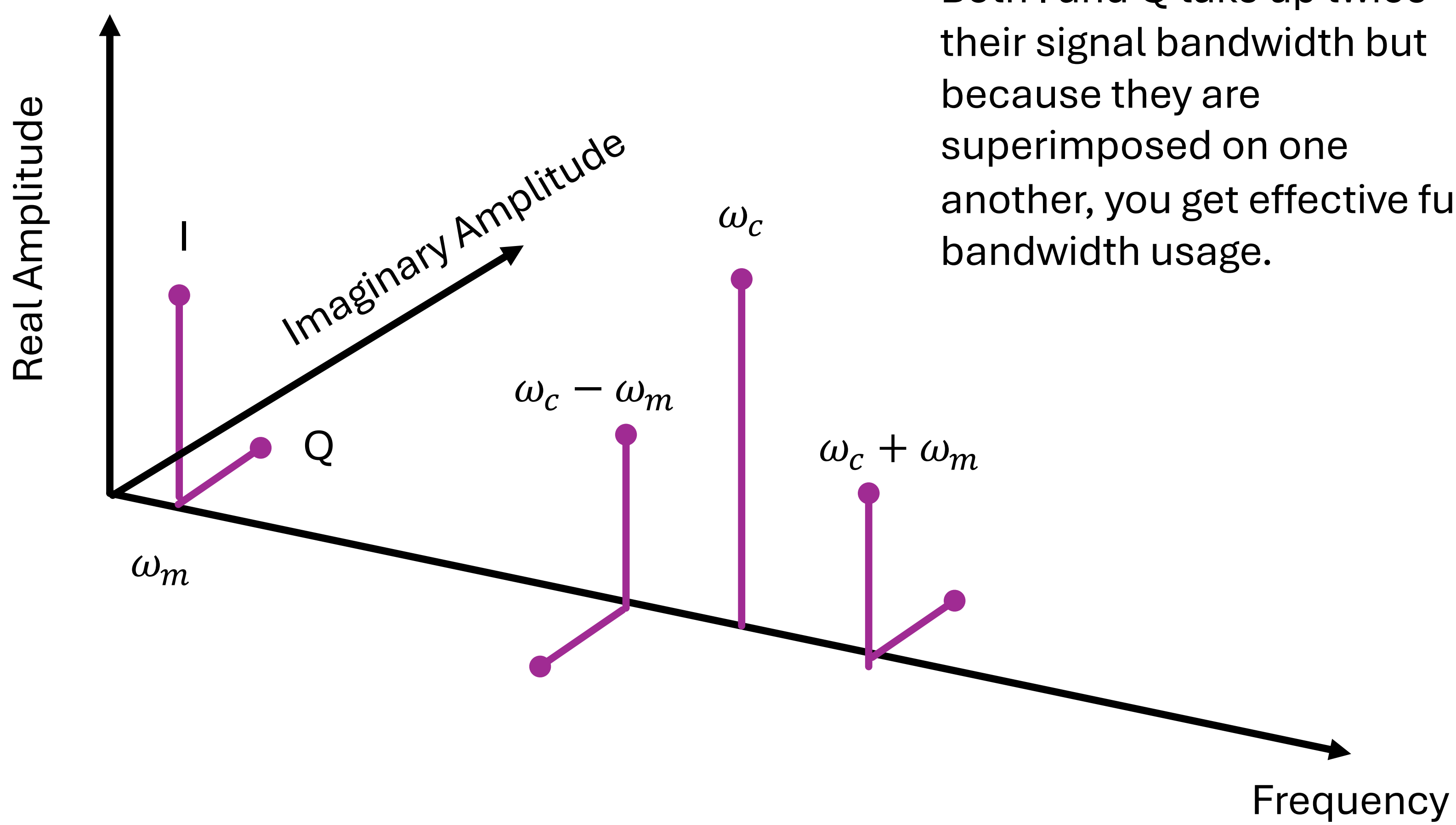
With I/Q Modulation…

- Now Modulating in Two-Dimensions



With I/Q Modulation...

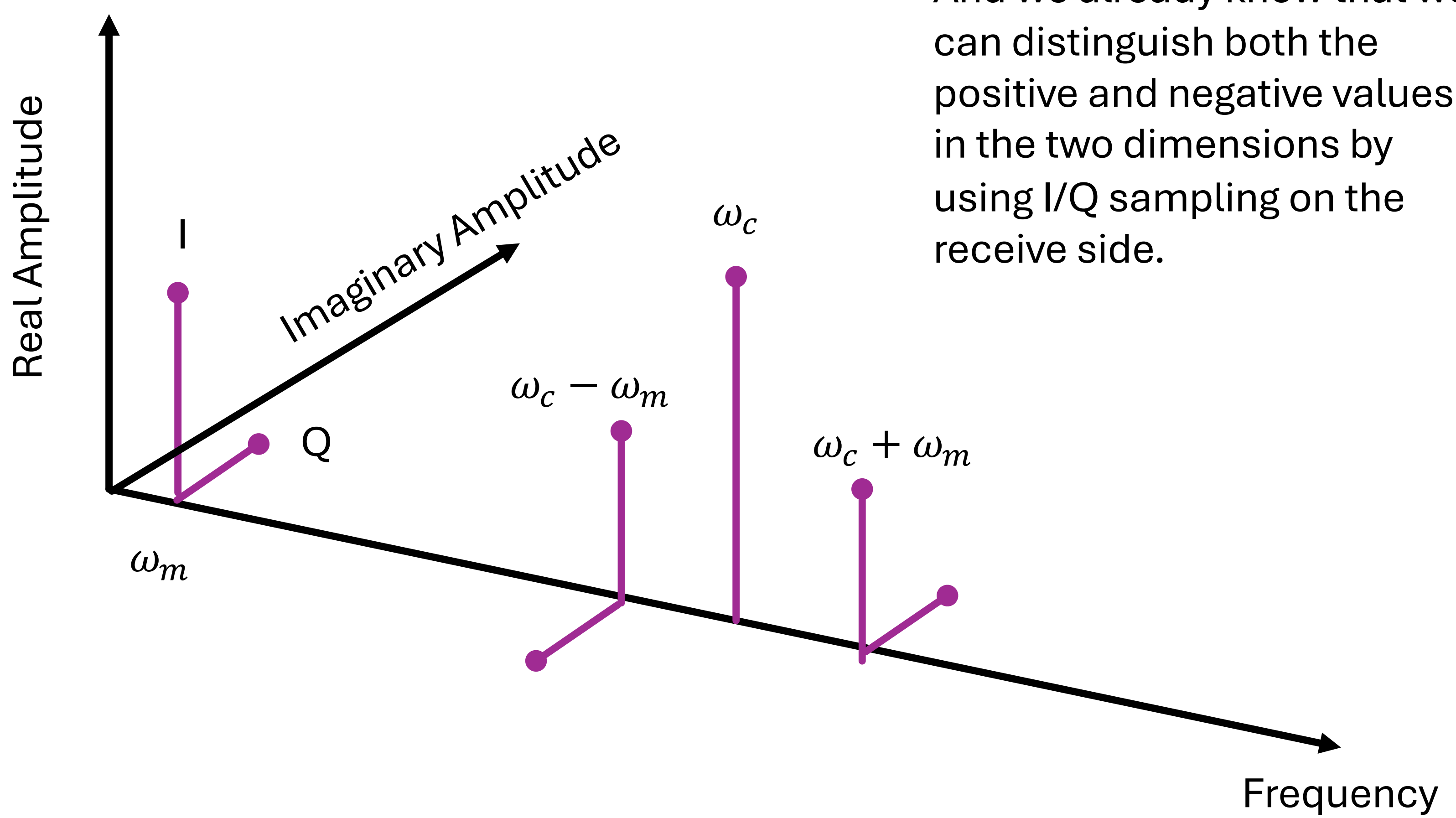
- There's now no redundant information being sent.



Both I and Q take up twice their signal bandwidth but because they are superimposed on one another, you get effective full bandwidth usage.

With I/Q Modulation…

- There's now no redundant information being sent.



And we already know that we can distinguish both the positive and negative values in the two dimensions by using I/Q sampling on the receive side.

I/Q signals

$$= \cos(\omega_c t) \cdot A(t)\cos(\phi(t)) - \sin(\omega_c t) \cdot A(t)\sin(\phi(t))$$

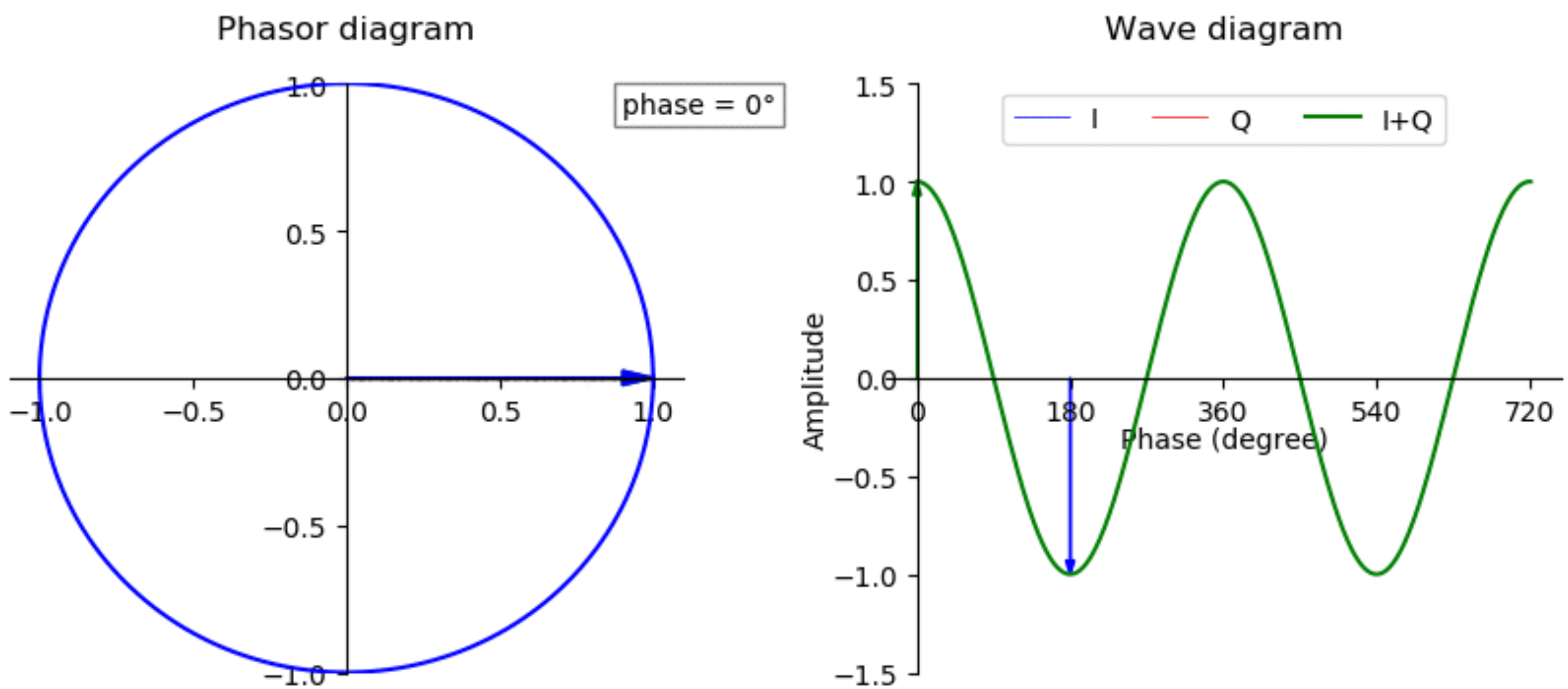
- You have two signals of fixed frequency and modulatable amplitudes.
- We don't need to think of those two amplitudes as related to the original signal, instead just think of them as modulatable signals on their own:

$$= \cos(\omega_c t) \cdot I(t) - \sin(\omega_c t) \cdot Q(t)$$

- This $I(t)$ and $Q(t)$ signal contain your information and can be useful in both creating and analyzing modulated signals.

Plot these I/Q values out on the complex plane…help us visualize signals and information…

quadrature component (y axis)

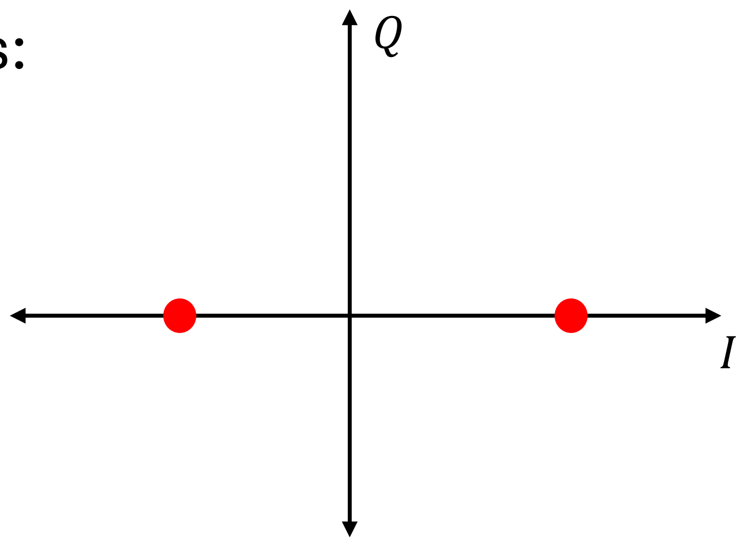


In-phase component (x axis)

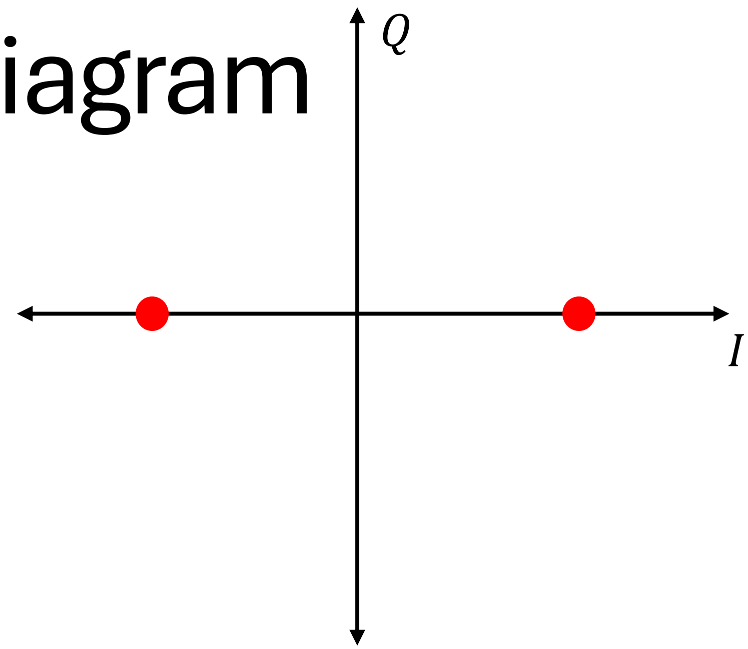
$$v(t) = \cos(\omega_c t) \cdot I(t) - \sin(\omega_c t) \cdot Q(t)$$

Varying I/Q

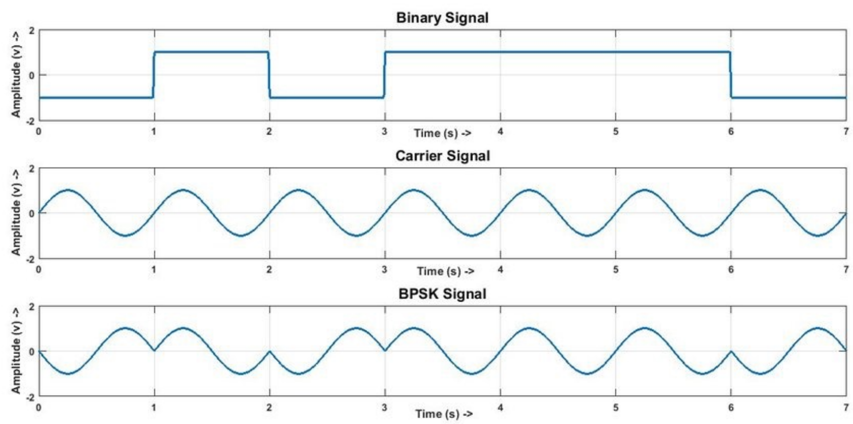
- Let's say we want to send binary 1's and 0's.
- One way to do this would be by doing the following:
  - $Q(t)$ is always set to 0
  - $I(t)$ is +1 for binary 1 and -1 for binary 0
  - We'd get an I-Q plot like this:



Constellation Plot/Diagram



- Plotting out the possible I/Q combinations in the 2D space is known as a constellation diagram
- Each constellation diagram is a way to depict all the forms of a signal that could be expected
- Time series plot of this signal shown here-→



https://www.researchgate.net/figure/Implementation-of-Binary-Phase-Shift-Keying_fig10_348364945

$$v(t) = \cos(\omega_c t) \cdot I(t) - \sin(\omega_c t) \cdot Q(t)$$

Binary Phase Shift Keying

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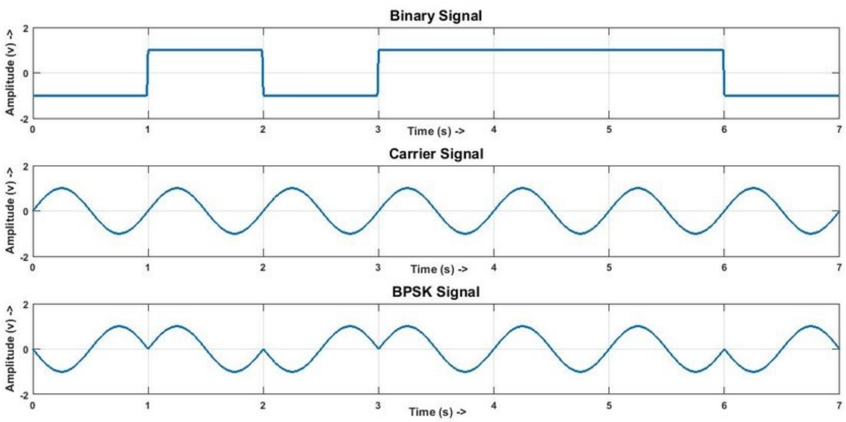
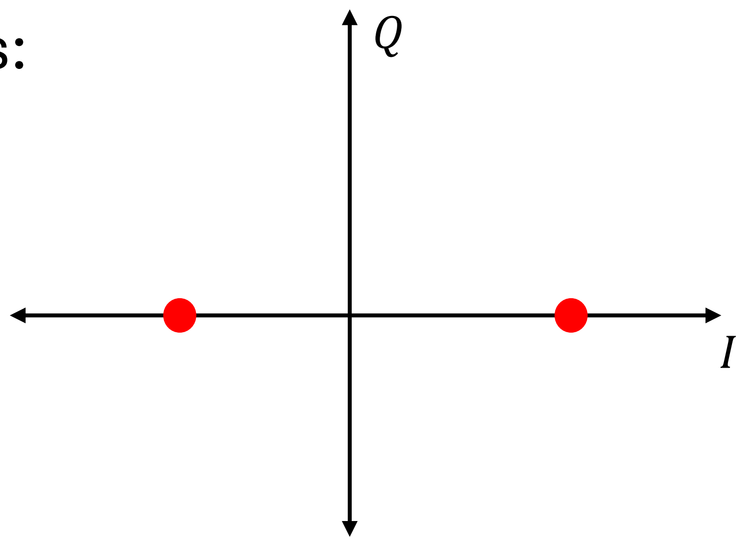




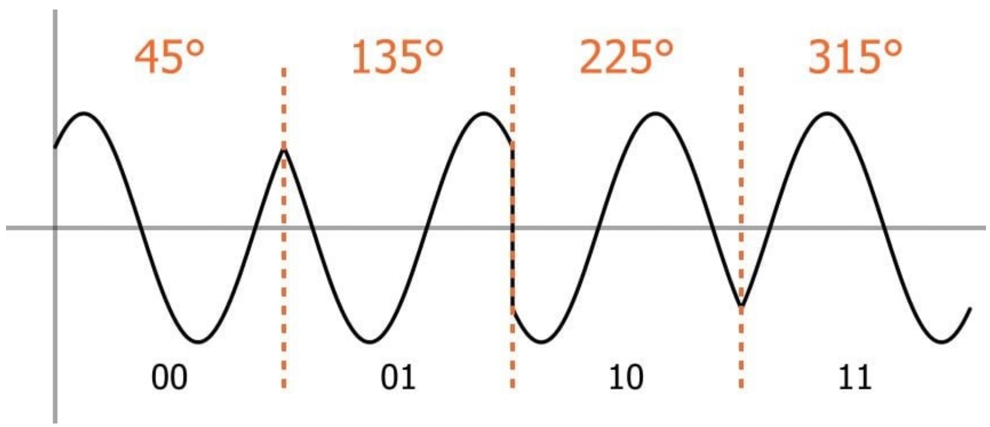
$$v(t) = \cos(\omega_c t) \cdot I(t) - \sin(\omega_c t) \cdot Q(t)$$

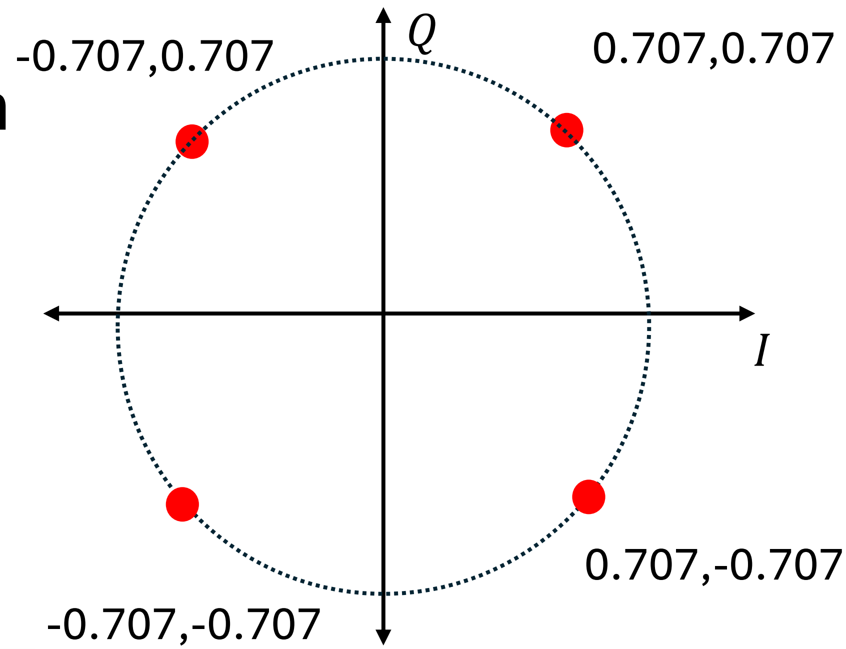
Can we do more?

- Right now, we have only two states, so can only send one bit per ”beat” of data.
- Could we do more? What if we use both dimensions of our I/Q signal?
- Have:
  - I be +0.707 or -0.707
  - Q be +0.707 or -0.707
- What would that constellation plot look like?

Quad-Phase Shift Keying

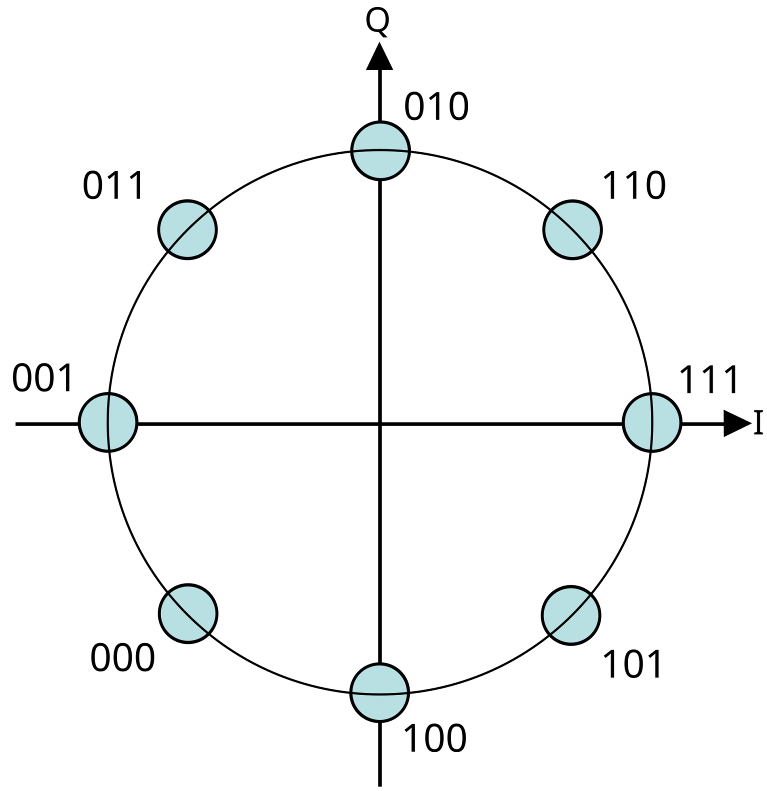
- Constant amplitude, but four distinct phases, each 90 degrees separated from eachother. Each measurement gives two bits of info





Keep going

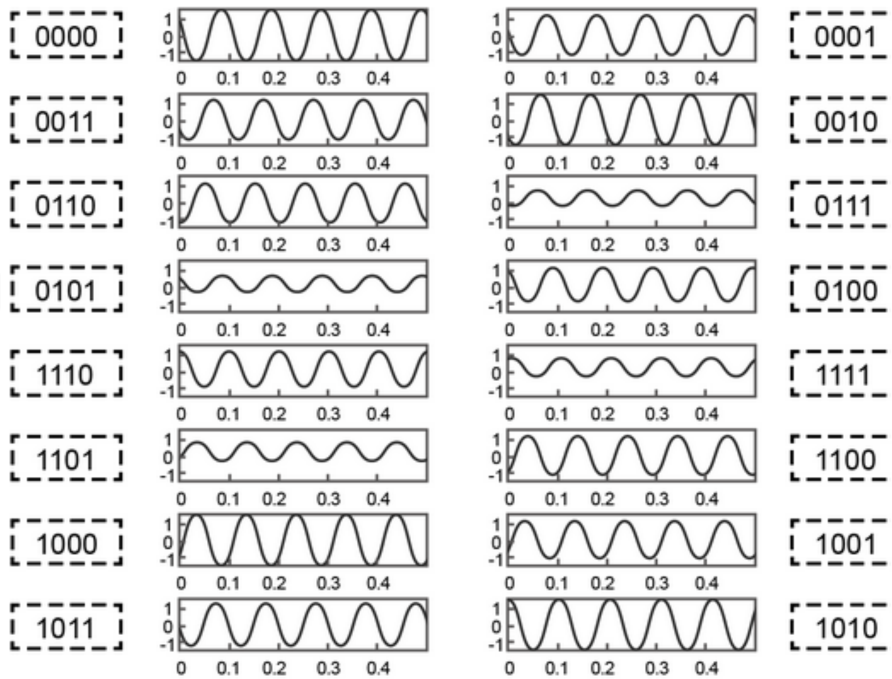
- Instead of assign I/Q separately, start to have pairs of I/Q values that position your signals all over the constellation plot as desired.
- 8 points separated by 45 degrees gives 8psk
- Each measurement gives 3 bits.



https://en.wikipedia.org/wiki/Constellation_diagram

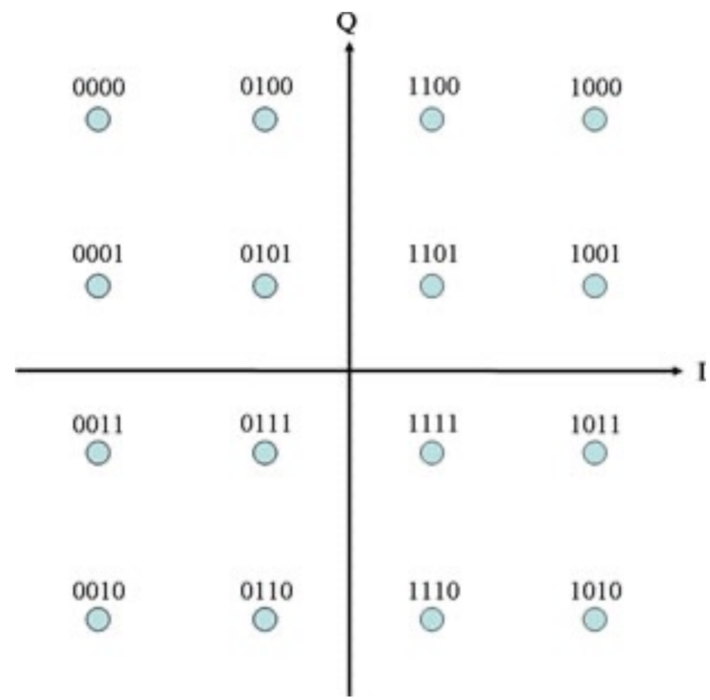
Can also not just stick to unit circle (vary amplitude of signal!

- Called Quadrature amplitude modulation.
- 16 QAM has 16 different possibilities:
- In time they look like:

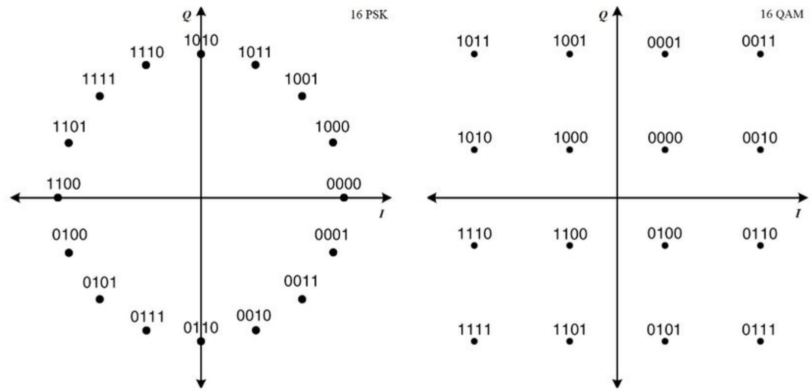


Can also not just stick to unit circle (vary amplitude of signal!

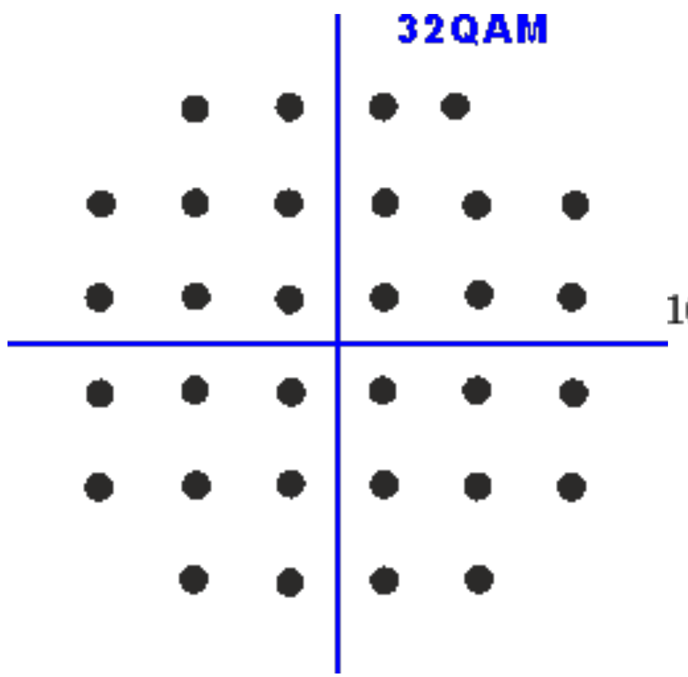
- Called Quadrature amplitude modulation.
- 16 QAM has 16 different possibilities:
- In constellation plot:

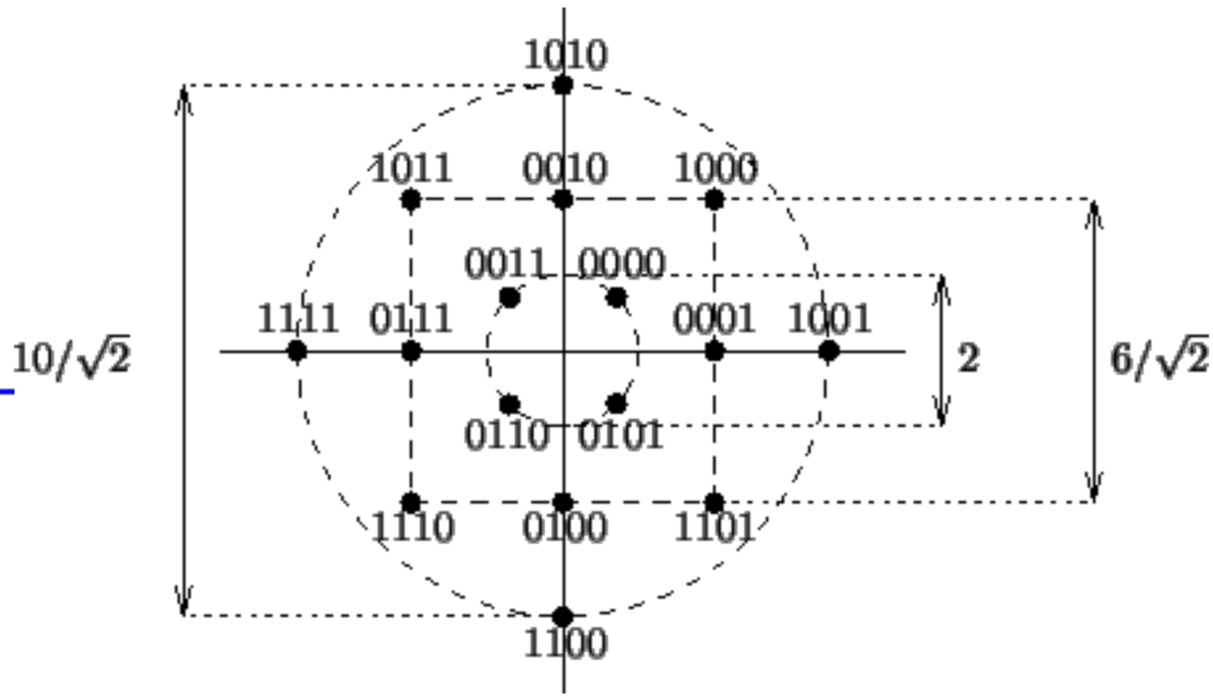


QAM vs. QPSK



- Both done using same toolchain
- QAM is more complicated to decode (uses amplitude and phase)/Re/Imag location in I/Q plane
- QPSK really only needs phase, once aligned

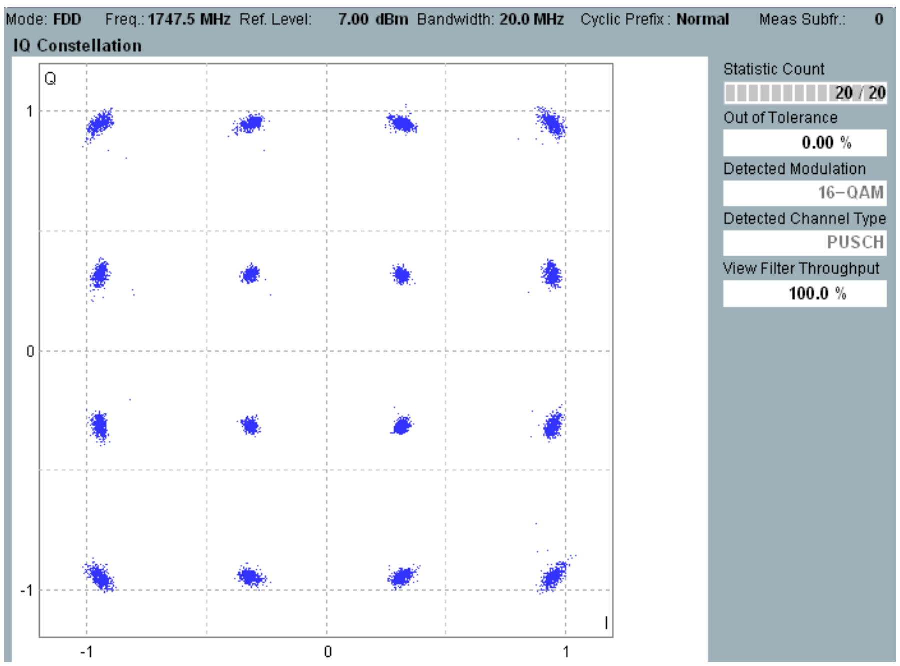




https://dsp.stackexchange.com/questions/31607/why-are-qam-constellations-regular-and-rectangular

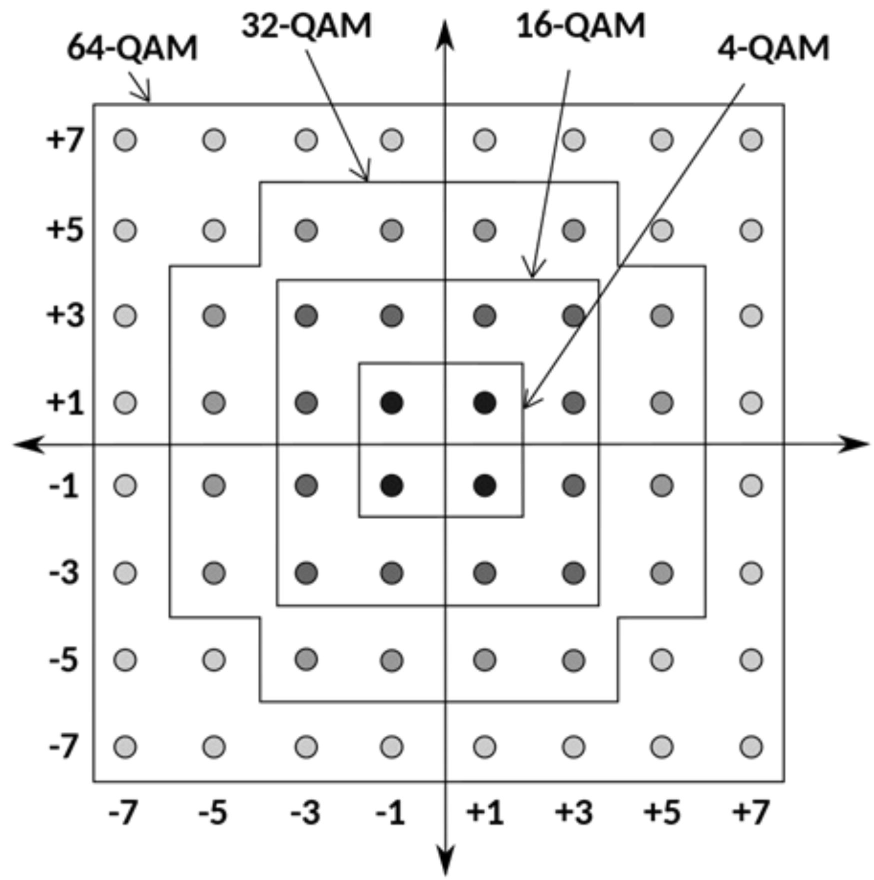
And this can keep going.

- This methodology of both modulation but also of analyzing signals can allow for relatively easy and productive analysis

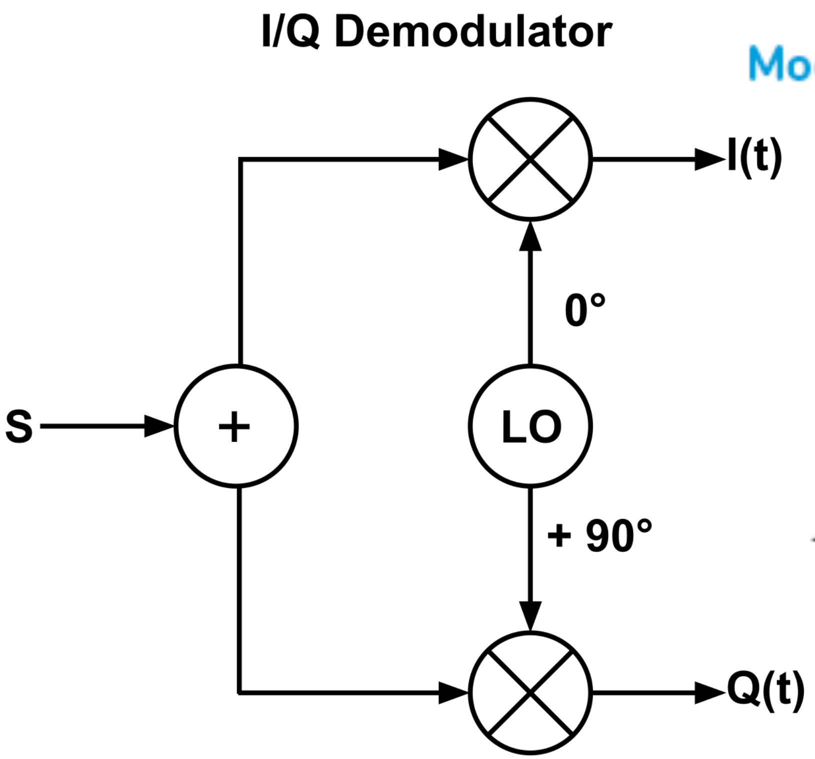


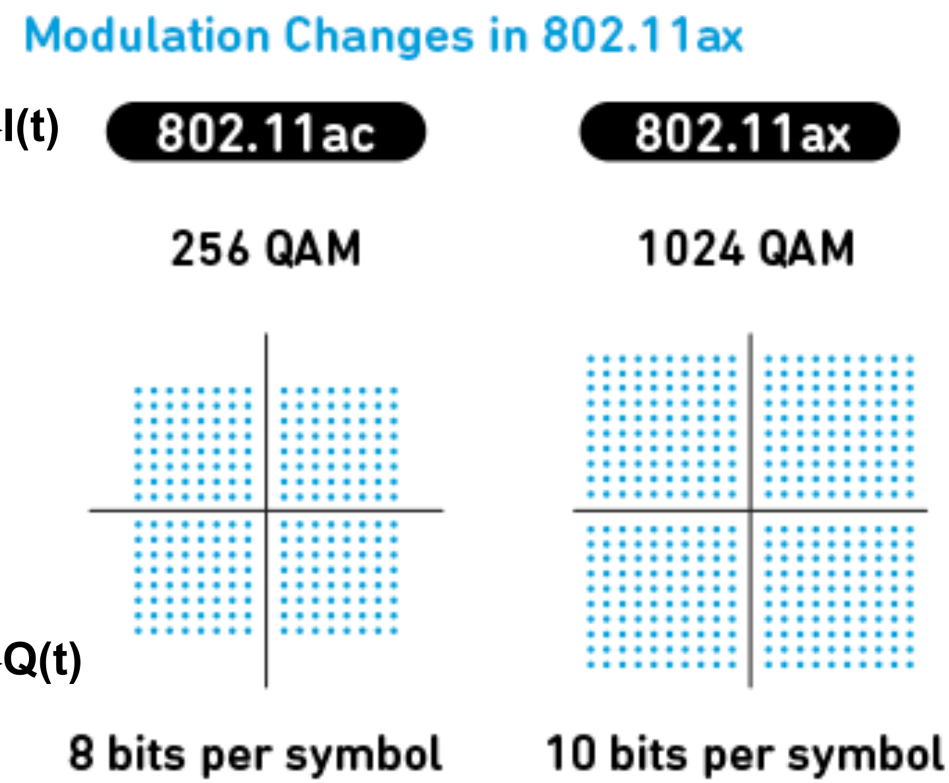
Wifi Uses QAM at higher and higher densities

- Each measurement becomes a symbol
-



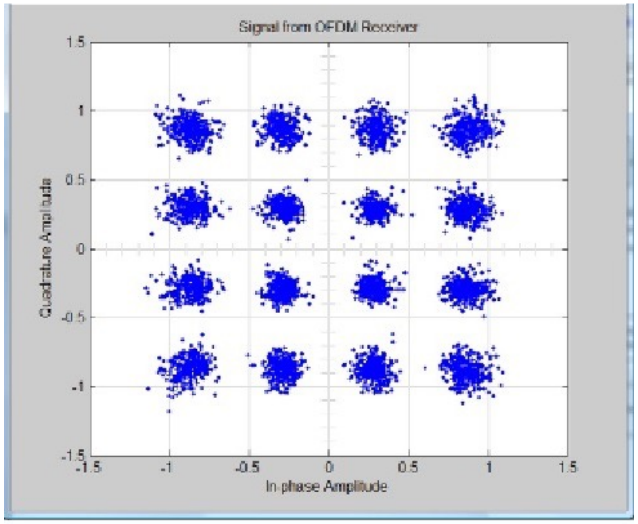
More advance wifi uses 256 and 1024 QAM



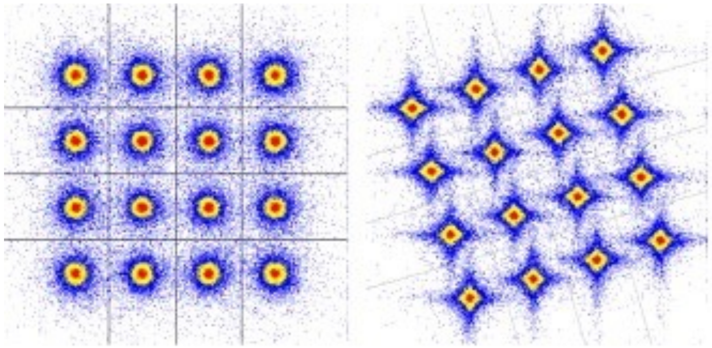


Get IQ values out

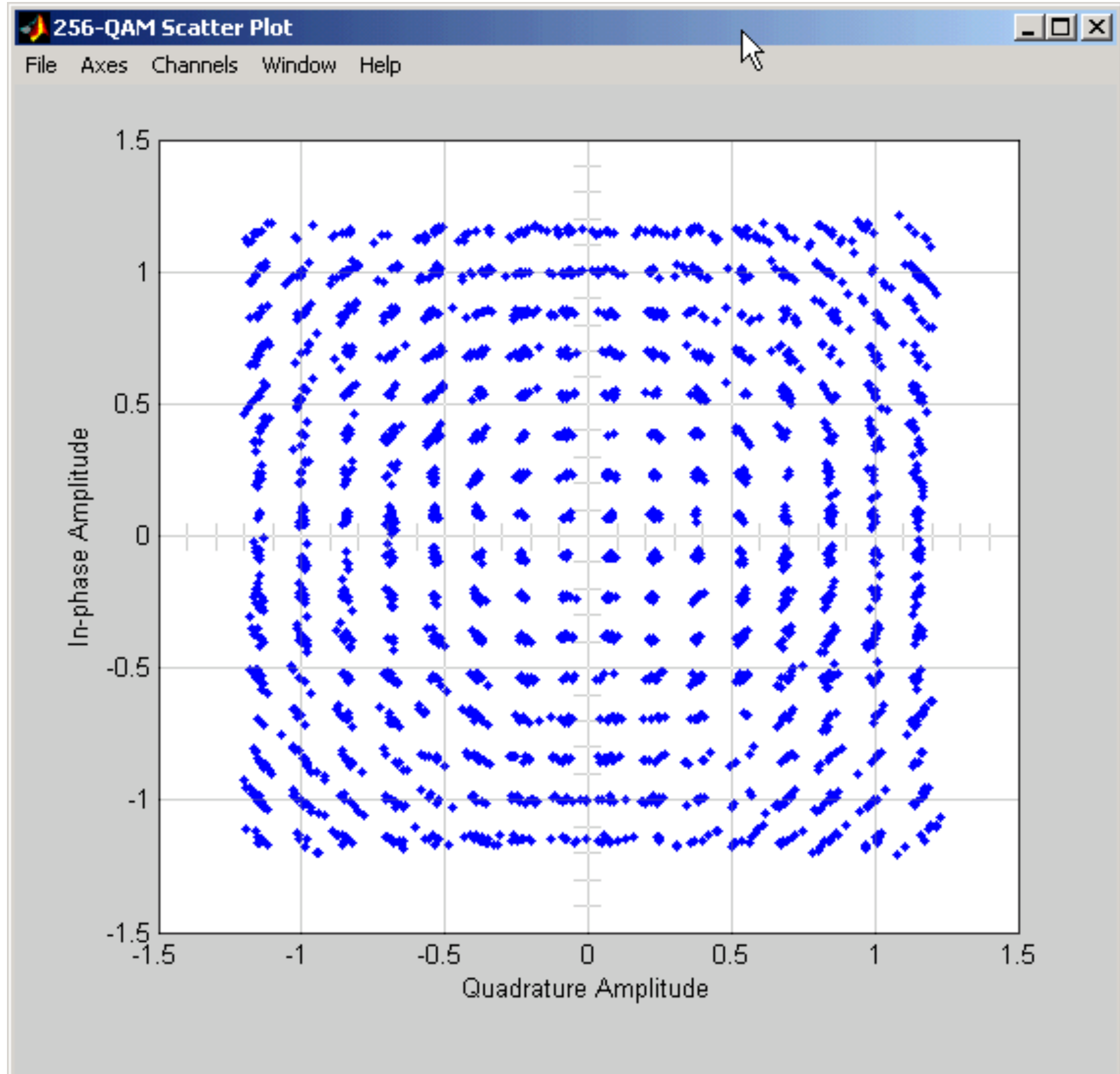
- From those extract meaning…



- Isn't always clean



https://www.analogictips.com/eye-and-constellation-diagrams-pt-2/



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